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DOI: 10.1177/0959651811422179
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What is This?
Robust trajectory modification for tip position tracking of flexible-link manipulators

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The manuscript was received on 20 June 2011 and was accepted after revision for publication on 11 August 2011.

DOI: 10.1177/0959651811422179

Abstract: This paper presents a composite controller for tip position tracking of flexible link manipulators. The main control challenge for flexible link manipulators is the non-minimum phase characteristics of the system. In this regard, an inner/outer control structure is proposed. As opposed to previous research in this area, the desired reference trajectory is robustly modified in an online scheme to minimize the tip tracking error utilizing the outer controller. The outer trajectory modifier is a $\mu$-synthesis based controller which modifies the reference trajectory of the inner loop in the uncertain situations. The inner loop controller is based on the Lyapunov redesign feedback linearization (LRFL) approach which is applied to alleviate the degrading effects of uncertainties and non-linearities presents in the dynamics of the flexible-link manipulator. In the inner loop, a conventional redefined output namely ‘close to the tip’ is considered to avoid the difficulties associated with the non-minimum phase behaviour of the main output (the tip). Conventional control strategies based on this choice of outputs lead to undesirable oscillations in the tip position. However, these oscillations are considerably minimized by applying the proposed outer loop trajectory modifier. Experimental and simulation results are presented to illustrate the significant improvements in tip tracking performance over the conventional methods.

Keywords: flexible-link, trajectory modification, robust feedback linearization, non-minimum phase systems, $\mu$-synthesis.

1 INTRODUCTION

Flexible link manipulators have received a great deal of interest during last decade. The exclusive characteristics of this structure are vital in some applications. For instance, in surgical robots, structural miniaturization is crucial to perform minimally invasive surgeries, which causes structural flexibility [1–5]. Furthermore, the use of flexible link manipulators results in superior energy efficiency and lower implementation cost as compared to rigid robots. Consequently, flexible manipulators are of interest in many current applications such as space industry [6–9]. However, modelling and tip position tracking of flexible links is more challenging as compared to rigid manipulators [10]. Unmodelled dynamics, non-linearities, and non-minimum phase behaviour make the precise tracking problem a challenging control concern. The instability of the zero dynamics associated with the tip position restricts the fascinating application of feedback linearization even where all non-linearities are known [11–13]. The non-minimum phase characteristic degrades the perfect tracking utilizing a causal controller [14, 15]. To overcome this problem, some modifications have been performed. In most cases, a new output is defined to guarantee the minimum phase behaviour of the system. In reference [16], the reflected tip was proposed as the output. In references [17–19] the authors considered the

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flexible manipulators as a singular perturbed system and redefined the output as a rigid part and a flexible part in the context of integral manifold theory. In reference [20] a point close to the tip was utilized, which has a superior performance as compared to earlier techniques. Furthermore, in reference [21] it was shown that the proximity of the new minimum phase output to the tip is linked to the payload. Consequently, when the payload is uncertain, the controller should be more conservative or complex. Other challenges are uncertainties caused by model truncation error, uncertain parameters, unmodelled dynamics, friction and backlash [22, 23].

In this paper, a composite controller is introduced for flexible link manipulators which guarantees small tip tracking error in uncertain situations. The composite approach consists of a μ synthesis controller as an outer loop plus a Lyapunov redesign feedback linearization (LRFL) technique as an inner control loop. Some researchers have focused on open loop input shaping to reduce the vibration of the tip [24, 25]. However, the performance and robustness of the open loop techniques are questionable when the dynamics of the flexible link are uncertain or time varying (since they are heavily based on the model resonance frequencies). Furthermore, even in ideal situations, utilizing the input shaping techniques the achieved performance suffers from the fact that the tip tracking issue remains intact and only the excitation of vibration modes are minimized.

In this paper a robust closed loop trajectory modifier (outer loop) is proposed which minimizes the tracking error of the main output (tip) utilizing online modification of the inner reference trajectory (based on the tip feedback). So, in opposition to previous conducted researches in this area, the desired trajectory is robustly modified in the outer controller. The main task of the inner loop is asymptotic stabilization of the redefined output in uncertain situations. In the literature the redefined output has been applied to deal with the non-minimum phase challenge of the main output (tip) in the issue of feedback linearization. However, utilizing the redefined output, considerable oscillations in the main output are generated. In this paper these oscillations are robustly alleviated and controlled in the proposed structure by online modification of the inner reference trajectory utilizing the outer μ controller. Input of the outer controller is the actual tip tracking error and the output signal is the modified desired trajectory for the redefined output, which is fed to the LRFL as the input of the inner loop. In conventional control strategies, it is vital to choose the redefined output as close as possible to the tip, to get small tracking error, whereas, in the present authors’ proposed control strategy this constraint is relaxed and the output can be selected inside a safe margin to ensure the stability of the zero dynamics in different situations, such as payload variation or uncertain payload and mass matrix. To employ the LRFL technique, an upper bound on the uncertainties is required. If this bound is conservatively designed, a high control effort is generated [26]. In this paper, an appropriate upper bound is proposed to alleviate this conservative behaviour. This bound is calculated online and utilized in the control strategy. The performance of the proposed technique is evaluated utilizing the oriented experimental and simulations results.

The rest of the paper is organized as follows. In section 2 the dynamic modelling and classic partial feedback linearization (PFL) are discussed. Section 3 introduces the structure of the proposed composite controller. Section 4 presents the simulation results. In section 5 the experimentally validation of the proposed technique is given and finally the paper is concluded in section 6.

2 DYNAMIC MODELLING AND FEEDBACK LINEARIZATION

Feedback linearization is a well-known non-linear controller utilized to control rigid manipulators, although it can not be employed for flexible links since the main output is non-minimum phase. Consequently, some sort of output redefinition should be applied to partially linearize the system. In this section, the dynamic modelling and conventional PFL of the flexible link manipulators are reviewed [11]. The dynamics of a flexible manipulator can be obtained using the Recursive Lagrangian approach [11, 12]

\[
M(\theta, \dot{\theta}) \begin{bmatrix} \ddot{\theta} \\ \ddot{\delta} \end{bmatrix} + \begin{bmatrix} f_1(\theta, \dot{\theta}) + h_1(\theta, \dot{\theta}, \delta, \ddot{\delta}) + F_1 \dot{\theta} + f_c \\ f_2(\theta, \dot{\theta}) + h_2(\theta, \dot{\theta}, \delta, \ddot{\delta}) + K \delta + F_2 \dot{\delta} \end{bmatrix} = \begin{bmatrix} u \\ 0 \end{bmatrix}
\]

(1)

where \( \theta \) is the hub angle, \( \delta \) is a \( n \times 1 \) vector of deflection variables, \( f_1, f_2, h_1 \) and \( h_2 \) are the terms due to Coriolis and centripetal forces, \( F_2 \) is the positive definite diagonal damping matrix related to the internal viscous friction, \( M \) is the positive definite mass matrix, \( K \) is the positive definite diagonal stiffness matrix and \( u \) is the hub torque. \( F_1 \) is the hub viscous friction and \( f_c \) is the Coulomb friction [27]. Now, define

\[
H(\theta, \delta) = M^{-1}(\theta, \delta) = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}
\]

(2)

then equation (1) can be re-written as
\[
\begin{bmatrix}
\dot{\theta} \\
\dot{\delta}
\end{bmatrix} = H(\theta, \delta) \begin{bmatrix}
u - f_1(\theta, \dot{\theta}) - h_1(\theta, \dot{\theta}, \delta, \dot{\delta}) - F_1 \dot{\theta} - f_2 \\
-f_2(\theta, \dot{\theta}) - h_2(\theta, \dot{\theta}, \delta, \dot{\delta}) - K_\delta \delta - F_2 \dot{\delta}
\end{bmatrix}
\]  

(3)

The output of the flexible link can be defined as [28, 11]

\[y = \theta + (W/l)\]  

(4)

where

\[W = \sum_i \varphi_i(x) \delta_i(t)\]  

(5)

In equations (4) and (5), ‘W’ denotes the deflection and ‘l’ denotes the link’s length also, \(\varphi_i(x)\) is a modal shape function related to the deflection variable \(\delta_i(t)\). A point close to the tip has been introduced in reference [20] and utilized in many researches [11, 12, 14, 21, 22]. This redefined output ‘\(y_{\text{redef}}\)’ is denoted by equation (6)

\[y_{\text{redef}} = y + \alpha(W/l)\]  

(6)

where \(-1 < \alpha < 1\)

In reference [20], it was shown that a positive critical \(\alpha^* < 1\) exists such that the zero dynamics is stable for \(-1 < \alpha < \alpha^*\). Moreover, in reference [22] it is illustrated that the \(\alpha^*\) increases by increasing the amount of the payload. Therefore, in variable or uncertain payload situation, ‘\(\alpha\’\) should be selected conservatively, which may cause considerable oscillations in the tip [11]. In this paper, an outer control loop has been applied to overcome these oscillations. The relative degree of this system is 2, so the dynamics associated with the redefined output has been derived as follows [11, 14]

\[\ddot{y}_{\text{redef}} = A(\theta, \dot{\theta}, \delta, \dot{\delta}) + B(\theta, \delta)\alpha \dot{u}\]  

(7)

where

\[A = -((H_{11} + \gamma \times H_{21}) \times (h_{11} + F_1 \times \dot{\theta} + f_{c_1}) - (H_{12} + \gamma \times H_{22}) \times (h_{12} + (F_2 \times \dot{\delta}) + K \times \delta)\]  

(8)

\[B = H_{11} + \gamma \times H_{21}\]  

(9)

In equations (8) and (9), \(\gamma = \alpha \times \dot{\theta}\) and \(\partial = \frac{1}{l}[\varphi_1 \ldots \varphi_n]\). Define a reference trajectory \(\ddot{y}_r, \dot{y}_r, \dot{y}_r\) for redefined output and also corresponding tracking errors \(\ddot{e} = \ddot{y}_r - \ddot{y}_{\text{redef}}, \dot{e} = \dot{y}_r - \dot{y}_{\text{redef}}\) and \(\dot{e} = y_r - y_{\text{redef}}\). Then, a feedback linearization law related to this output can be achieved via equation (10) [11, 21]

\[u = B^{-1}[-A + v]\]  

(10)

where

\[v = \ddot{y}_r - K_1 \dot{e} - K_2 e\]  

(11)

Employing equation (10), the linearized output is achieved as equation (12)

\[\ddot{y}_{\text{redef}} = v\]  

(12)

By substituting equations (10) and (11) into equation (7), and assuming \(K_1, K_2 > 0\), a Hurwitz linear dynamic of the redefined output tracking error will be achieved via equation (13)

\[\ddot{e} + K_1 \dot{e} + K_2 e = 0\]  

(13)

Note that, in conventional control strategies \(\ddot{y}_r = y\), and the controller makes the redefined output to track a reference trajectory. So the tip can approximately tracks the desired trajectory in an open loop approach with considerable oscillations so the performance is directly linked to the closeness of the redefined output to the tip. However, the approach presented in this paper assumes that \(\ddot{y}_r \neq y_r\) and the reference trajectory will be modified to achieve a superior tip tracking performance in a closed outer loop structure.

3 THE PROPOSED ROBUST COMPOSITE CONTROLLER

In this section, the main parts of the proposed composite technique are clarified. First, the idea of the robust feedback linearization, as the inner loop, is given which is utilized to ensure the asymptotic tracking of the redefined output. Then, the \(\mu\) synthesis controller is introduced as the outer controller to design the reference trajectory for the inner controller. The input of the outer controller is the tip tracking error and its output is the reference trajectory for the inner controller. Figure 1 illustrates the main structure of the proposed technique.

3.1 Robust feedback linearization

As mentioned, the main assumption of feedback linearization is the availability of an exact model. But the precise modelling is not achievable via phenomena such as: backlash, parameter uncertainties, friction and un-modelled dynamics. When the model is not accurate, there is no convergence guarantee for feedback linearized system, so a robust technique, such as sliding mode control, is vital [26]. In this paper the LRFL is utilized and proposed as a promising technique to control the redefined output. Considering equation (10), suppose that, the present
authors’ nominal knowledge about A and B, is represented by $\hat{A}$ and $\hat{B}$ and the mismatched errors are represented by $\hat{A}=A-\hat{A}$ and $\hat{B}=B-\hat{B}$, so equation (12) can be rewritten as follows, in equations (14) to (17)

$$u = \hat{B}^{-1}[-\hat{A} + v] \rightarrow \tilde{y}_{\text{ref}} = A - B\hat{B}^{-1}\hat{A} + \hat{B}\hat{B}^{-1}v$$

$$\rightarrow \tilde{y}_{\text{ref}} = (A - B\hat{B}^{-1}\hat{A}) + (\hat{B}\hat{B}^{-1}v)$$

$$\rightarrow \tilde{y}_{\text{ref}} = (A - B\hat{B}^{-1}\hat{A}) + (\hat{B}\hat{B}^{-1}v) + v$$

Comparing equations (17) and (12), the additional term $\psi(v, \hat{\theta}, \theta, q_1, q_2, \dot{q}_1, \dot{q}_2) = (A - B\hat{B}^{-1}\hat{A}) + (\hat{B}\hat{B}^{-1}v)$ in equation (17) is an uncertainty produced by $\hat{A}$ and $\hat{B}$. Consider a new linearization law presented in equation (18)

$$v = \tilde{y} - K_1\dot{e} - K_2\dot{e} + \sigma$$

In equation (18), ‘$\sigma$’ is an additional term, designed to eliminate the potential destabilizing impact of ‘$\psi$’. Let $E = [e, \dot{e}]^T$ be defined. The dynamics of the error is defined as

$$\dot{E} = A_e E + B_e (\sigma + \psi)$$

where

$$A_e = \begin{bmatrix} 0 & 1 \\ -K_2 & -K_1 \end{bmatrix}, \quad B_e = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Let an upper bound on the uncertainty $\psi$ be defined as follows

$$\psi < \rho(E, t)$$

And also define a Lyapunov function $V = E^T P E$. In [26] it has been illustrated that for LRFL, ‘$\sigma$’ can be designed via a ‘continuous’ method, represented in equation (22) to satisfy $\dot{V} < 0$

$$\sigma = \begin{cases} -\rho(E, t) B_e^T P E & \text{if } B_e^T P E > \epsilon \\ -\rho(E, t) \epsilon^T B_e P E & \text{if } B_e^T P E \leq \epsilon \end{cases}$$

In equation (22), ‘$\epsilon$’ is an arbitrary small value and ‘$P$’ is a positive definite matrix satisfying the Lyapunov equation (23)

$$A^T P + PA = -Q, \quad Q > 0$$

It has been proved [26] that, utilizing the LRFL, presented in equations (18) and (22), all trajectories of the system in equation (19) are uniformly ultimately bounded. Moreover the challenge of demonstrating the upper bound, ‘$\rho$’, is a vital issue. A conservative choice, leads to generate an undesirable control effort. So in this paper a reasonable upper bound is calculated online (for single-link) to alleviate this conservative behaviour. The mentioned bound is achieved, utilizing the upper and lower bounds on the link’s parameters. The calculation is illustrated in the Appendix.

### 3.2 Online inner trajectory modification via $\mu$ synthesis

Applying the proposed LRFL, dynamics of the main output is partially linearized and degrading effects of uncertainties are alleviated. So the redefined output robustly tracks its corresponding desired trajectory. Afterwards, a $\mu$ controller is employed as the outer loop to ensure robust performance (RP) for tip tracking by modifying input of the inner-loop. In conventional output redefinition techniques, to achieve an acceptable tracking error, an output very close to the tip was required [22]. It has been shown that closeness of the redefined output to the tip is restricted via the payload [21]. As a result, when payload is unknown or varying, a conservative choice should be performed, which degrades the performance. In the proposed technique, unlike all conventional approaches, it is not vital to have a redefined output very near to the tip (e.g., when $\alpha \approx 0.3$), and it can be selected in a safe margin to ensure a minimum phase redefined output. Consequently, to minimize the tip tracking error, a
μ synthesis controller is applied for online modifying the inner loop input. First, the remaining nominal dynamics is linearized around its equilibrium point. Then, the representation of the uncertain model is derived and the control requirements are transformed to some frequency based weighting functions. Finally, a D–K iteration technique is utilized to design the outer controller [29]. In this paper a flexible link manipulator with the parameters listed in Table 1 is considered.

Note that the parameters given in Table 1 belong to an experimental set-up presented in reference [11]. In Table 1, \( l \) is the length of the link, \( \gamma \) is the mass per unit length, \( h_i \) is hub inertia, \( b \) is the hub’s viscous friction, \( E \) is Young’s modulus, \( I \) is the beam area moment of inertia, \( v \) is the \( j \)th resonance frequency, \( c_i \)’s are the viscous damping coefficient, and \( M_f \) is the payload. As mentioned, after applying the LRFL, new ‘tip dynamics’ are derived and linearized around its equilibrium point. Considering equations (10) and (11) the inputs of the feedback linearized system are \( b_yr \) and its first and second derivatives and output is the tip. Based on the actual parameters given in Table 1, the transfer function between the tip and \( b_yr \) can be obtained as

\[
G = \frac{0.21574(s^2 + 13.425s + 125)(s^2 - 11.6s + 147.3)}{(s^2 + 0.349s + 15.2)(s^2 + 1.52s + 261.1)} \tag{24}
\]

Consider equation (24); the mentioned flexible link has two complexes right half plane (RHP) zeros \((z_1, z_2)\) on \(5.79 \pm 10.67i\). Note that, the nominal uncertain transfer function is presented in the simulations part in equation (31). After deriving the nominal linear approximation, uncertain control structure is developed as Fig. 4. In this structure the exogenous signal is reference trajectory \( (y_{ref}) \), the regulated output is weighted tip tracking error \( (Er) \), the control signal is inner trajectory \( (\hat{y}_r) \) and the measured output is tip tracking error \( (err) \). As mentioned before, the applied LRFL compensates some considerable fractions of the existing uncertainties and non-linearities. Although parametric uncertainties, neglected higher frequency modes and the difference between the approximated linear model \((\hat{G})\) and real behaviour between the tip and \( \hat{y}_r \), expose some uncertainties, which should be contemplated in the design. Two alternatives are evaluated to model the uncertainties: the additive and the input multiplicative models. Figures 2 and 3 illustrate the magnitude bode diagrams of the uncertainties related to multiplicative and additive model respectively for some samples. These results are achieved by altering \( M, K, F_1, F_2, f_c \) around their nominal values. Comparing Figs 2 and 3 it is clear that the input multiplicative model has a superior behaviour in representing the uncertainties due to the less amplitude especially near to the low operational frequencies. Consequently the multiplicative model is selected.

The interconnection structure can be represented in Fig. 4. The main goal of the \( \mu \) controller is to

<table>
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<th>Table 1 Applied parameters</th>
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**Fig. 2** The bode diagram related to the multiplicative uncertainty

**Fig. 3** The bode diagram related to the Additive uncertainty

**Fig. 4** Representing the system uncertainties
minimize the effects, exposed by \( y_{ref} \) on the weighted tracking error, \( Er \) in a RP manner. The weights \( W_{er} \), \( W_{ref} \), and \( W_{delta} \) should be designed to achieve the desired performance. The proposed weighting functions for the tip tracking issue are designed as follows.

3.2.1 The proposed \( W_{er} \)
The first resonance frequency of the supposed flexible manipulator is about 3 rad/s, so the operational bandwidth is restricted in low frequencies. As a result the performance weighting function \( W_{er} \) is taken to be a low pass filter as equation (25)

\[
W_{er} = \frac{0.8}{s + 0.02}
\]  

Equation (25) shows that the interested frequencies (for minimizing the \( Er \)) are assumed to be relatively less than 1.5 rad/s. Moreover the high DC gain is designed to minimize the tracking error in low frequencies and steady state.

3.2.2 The proposed \( W_{ref} \)
‘\( W_{ref} \)’ defines the frequency content of the desired trajectories. It was assumed to be less than 10 rad/s which is a reasonable value especially in practical applications [4, 14]. Consequently \( W_{ref} \) is taken to be a low pass filter as equation (26)

\[
W_{ref} = \frac{10}{s + 10}
\]  

3.2.3 The proposed \( W_{delta} \)
As mentioned, the uncertainties are assumed to belong to a set of input multiplicative model, as equation (27)

\[
G = \hat{G}(1 + W_{delta}\Delta)
\]  

Where ‘\( \hat{G} \)’ is nominal model and \( \Delta \) is normalized uncertainty satisfying \( \Delta_x = 1 \). Consequently \( W_{delta} \) should satisfy

\[
|W_{delta}| > \frac{G}{G} - 1
\]  

Equation (28) should be held for all ‘\( G \)’ computed by altering the link’s parameters in the permissible ranges around the related nominal values. As mentioned, the result has been shown in Fig. 2 when \( M, K, F_1, F_2, f_c \) are uncertain. Note that, in high frequencies, the validation of the assumed linear model is degraded by exciting the higher vibration modes, so the gain of \( W_{delta} \) should be relatively large in high frequency to reject this kind of uncertainties. Consequently \( W_{delta} \) is designed as equation (29)

\[
W_{delta} = \frac{8.33(s + 0.3)(s + 60)}{(s + 1.5)(s + 400)}
\]  

Utilizing the D–K iteration method, the controller is designed with the upper bound 0.88 for \( \mu \) which ensures the ‘robust performance’ in the defined situation [29].

In this section the proposed robust architecture including a robust inner loop in addition to a robust outer modifier is proposed for enhancing the performance of the classical techniques. There are some remarks which should be mentioned as follows

Remark 1
In comparison with classical techniques no more facilities, equipment, or measurement is required for the proposed architecture. As mentioned before, the inner loop is a robust format of the classic PFL and the outer loop is a robust position controller which is utilized as the trajectory modifier. In order to ensure the convergence of the system in uncertain situations, upper and lower bonds on the uncertain values are required.

Remark 2
In order to measure the link deflection, in practical applications, strain gauges are one of the best alternatives which are usually attached on the flexible link body near to the hub (to maximize the sensitivity of the sensor). Note that, utilizing the measured deflection, both actual output and redefined output can be easily calculated (considering equations (4) and (6)). Consequently, no more measurement is required in the proposed technique in comparison with classical methods, such as PFL controllers or Singular Perturbation-based approaches [4, 9]. In addition to the above, it should be noted that, similar to most classical methods, only the first derivative is required in order to implement the inner loop controller. And the outer loop modifier just utilizes the actual tip position of the flexible link.

4 NUMERICAL SIMULATIONS

In this part, the simulation results are presented to demonstrate the effectiveness of the proposed method. First, in case (a) the inner controller is evaluated, then in case (b) the entire control system is considered and the achieved performance is evaluated in a considerably uncertain situation.
Afterwards, in simulation (c), the effectiveness of the proposed modifier is evaluated when the challenge of uncertain mass matrix does not exist. Next, in case (d) the performance of the proposed composite technique is compared with the classic PFL controller in uncertain situations. Then, in case (e), the tracking issue is studied and a comparison is made between the proposed technique and the classic PFL method, when the reference trajectory is a sinusoidal signal. Finally, in simulation (f) the performance and stability of simple classic PD controller is simulated in order to evaluate the capability of the proposed technique in comparison with simple classical methods.

4.1 Inner loop controller

In case (a), the efficiency of the LRFL (Inner controller) is evaluated to control the redefined output. First, assume $M$, $K$, $F_1$, and $f_c$ as the real parameters considering the definitions given in Table 1, additionally assume $\hat{M}$, $\hat{K}$, $\hat{F}_1$, $\hat{F}_2$, and $\hat{f}_c$ as the matrices and values, represent the present authors' nominal knowledge about the parameters. In this simulation, the actual relation between the nominal parameters and the real parameters are $\hat{M} = M(1+5\%)$, $\hat{K} = K(1+5\%)$, $\hat{F}_1 = F_1(1+3\%)$, $\hat{F}_2 = F_2(1+3\%)$ a $\hat{f}_c = f_c(1+5\%)$. Additionally the present authors' knowledge upon the upper and lower bounds is denoted by

$$M \in \hat{M}(1\pm10\%), K \in \hat{K}(1\pm10\%),$$
$$F_1 \in \hat{F}_1(1\pm5\%), F_2 \in \hat{F}_2(1\pm5\%), f_c \in \hat{f}_c(1\pm10\%)$$  \hspace{1cm} (30)

$$\hat{G} = \frac{0.215(s^2 + 13.2s + 121.8)}{(s^2 + 0.332s + 14.7)(s^2 + 1.44s + 254)}$$  \hspace{1cm} (31)

Figure 5 represents the step response of the redefined output applying the LRFL (only). Note that in these simulations $\alpha = 0.3$ which is relatively small to ensure a minimum phase redefined output in presents of uncertainties on the mass matrix and payload. Figure 6 represents the degrading effect of this high level uncertainty on the performance of the conventional PFL. Figure 7(a) represents the proposed upper bound on the uncertainties comparing with the real uncertainties (which is available in the simulation) when the LRFL is applied. It can be inferred by comparing Figs 5 and 6 that the proposed LRFL exhibits a superior performance in uncertain situations additionally the un-modelled dynamics extremely degrade the performance of the conventional feedback linearization. So, utilizing the

![Fig. 5](image1.png) The redefined output response to a 0.2 rad. step input. $\alpha = 0.3$, $K_1 = 7$, $K_2 = 12$ (only the LRFL is employed)

![Fig. 6](image2.png) The redefined output response to a 0.2 rad step input (only the conventional PFL is employed)

![Fig. 7](image3.png) (a) The calculated upper bound comparing with the real uncertainty (only the LRFL is employed); (b) The control effort generated by LRFL, $\alpha = 0.3$, $K_1 = 7$, $K_2 = 12$ and the input is a 0.2 rad step (only the LRFL is employed)
LRFL method, the concern of exact modelling will be alleviated. In Fig. 7(a) it has been shown that, the proposed upper bound on the uncertainty is reasonable and the amplitude of the upper bound converges to zero when the real uncertainty converges to zero. Note that the conservativeness of the technique is linked to the calculated upper bound. The calculations are illustrated in the Appendix. Consequently, according to the considerable uncertainties in this simulation, especially in the early instances, when the real uncertainty is extremely high and as a result the upper bound is high as well, a conservative control effort is generated, which can be seen in Fig. 7(b). If the upper bound is designed in a more conservative way, the control effort would be undesirably magnified.

4.2 Composite controller

In case (b) performance of the composite controller for minimizing the tip position tracking error is evaluated. First, tip position behaviour is simulated employing only LRFL, where the outer controller does not exist. The step response is illustrated in Fig. 8. Afterwards, the outer controller is applied to minimize the tip tracking error. The result is shown in Fig. 9. Uncertain situation is the same as the previous simulation. Figure 10 compares the outer control signal and the redefined output profile. As mentioned, the outer control signal is the modified trajectory which has been utilized as the reference trajectory of the inner controller. In Fig. 11 the applied torque is shown. Similar to Fig. 7(b), conservative control effort is generated in early instances, since the amplitude of the real uncertainty and its upper bound is considerable. Comparing Figs 8 and 9, it is obvious that, the outer controller extremely improves the performance of the system and the \( \mu \) controller is able to minimize the error between the tip position and the reference trajectory which is a 0.2 rad step. In Fig. 8 considerable oscillation is caused by the smallness of the ‘\( a \)’ when the outer loop is ignored. Note that, applying the proposed trajectory modifier, it is not vital to select the redefined output very close to the tip for ensuring small tip tracking error, and it can be selected inside a safe margin to ensure the stability of the zero dynamics, especially when the mass matrix or payload is uncertain. In this simulation \( a \) is considered relatively small (\( a = 0.3 \)) since the mass matrix suffers from considerable uncertainties. Figure 9 exhibits the outstanding capability of the proposed method for precise tip tracking utilizing the idea of online trajectory modification. In Fig. 10 the modified trajectory is compared with the redefined output which exhibits an acceptable tracking error and results in a small tracking error between the tip and the main reference trajectory.

4.3 The case of known mass matrix

In the previous simulations the abilities of the proposed technique is illustrated. As mentioned before,
in the literature it has been shown that, the permitted $a$ is directly linked to the payload, consequently when the challenge of the uncertain mass matrix exists, the redefined output should be selected in a safe margin which degrades the performance of the conventional techniques. It is shown that utilizing the proposed method, the performance is considerably enhanced. In the previous simulations in order to highlight the aforementioned feature, it is assumed that $a=0.3$, which is an extreme conservative small value. In this simulation in order to show the abilities of the designed trajectory modifier, the performance is evaluated when the challenge of uncertain mass matrix does not exist ($M=M$). Consequently a larger $a$ can be selected. In this simulation, $a=0.7$. The results are shown in Figs 12 and 13.

Comparing Figs 12 and 9 utilizing the proposed modifier, when $a=0.7$, the rise time of the tip response is incredibly increased and the vibrations are rapidly damped. The reason is that by increasing $a$, more non-linearity is eliminated and the uncertainties are considerably alleviated so the loop gain can be more magnified utilizing the robust modifier and the performance is enhanced as the result. If the mentioned modifier is not utilized the tip response would suffers from undesirable oscillations as shown in Fig. 13.

Comparing Figs 13 and 8, since a larger $a$ is considered in this simulation, the response contains less oscillation. Considering Figs 12 and 13, it can be inferred that the proposed robust modifier, considerably decreases the vibration and enhances the performance of the tip position tracking.

4.4 The comparison with classic PFL in uncertain situation

In this part the performance of the classic PFL is simulated to achieve a comparison with the proposed composite technique. The uncertain situation is similar to the case (c). The result of employing classic PFL technique is given in Fig. 14.

Comparing Figs 12 and 14, the superior performance of the proposed composite technique is obvious. The robust trajectory modifier exhibits a considerable improving role in alleviating the vibrations and the effects of the uncertainties.
4.5 Sinusoid reference trajectory-tracking comparison

In this simulation the performance of the proposed technique is compared with the classic PFL when the reference trajectory is a sinusoidal signal. The amplitude of the reference signal is 0.05 rad and the frequency is 1 rad/s. The uncertain situation is similar to the case (c). The results are shown in Figs 15 and 16. Where Fig. 15 exhibits the superior performance of the proposed technique and Fig. 16 shows the undesirable behaviour of the classic method.

Comparing Figs 15 and 16 it is obvious that the proposed composite approach is considerably more effective in dealing with the tracking problem of the flexible link in uncertain situations.

In addition to the above, the control effort of the proposed technique and the comparison between the real and calculated upper bound on the uncertainty is given in Figs 17 and 18. As has been discussed before, since the amplitude of the uncertainty is relatively large in early instances, the control effort is magnified. In Figs 17 and 18 this is highlighted via zoomed graphs. Also comparing Figs 17, 11, and 7, since the existing uncertainty related to simulation (e) is the least, so the high amplitude transient behaviour shown in Fig. 17, is less than the other cases.

4.6 Comparison with simple PD controller

In order to evaluate the capability of the proposed composite architecture in comparison with classical controllers, in this simulation, the simple PD controller is utilized to cope with the challenge of tip tracking control of flexible link manipulators. It is clear that the ability of classic PFL (classic advanced method) is higher than the classic PD controller since the existing non-linearities are alleviated in the classic advanced method. However the implementation of the PD controller is considerably easier in comparison with the PFL technique. The performance and stability of PD controllers are heavily dependent on the gain adjustment since the existing non-linearities significantly degrades the performance and stability of the system. In this section the mentioned adjustment is performed by hand-fine-tuning in order to achieve acceptable
performances in two different situations. The results are shown in Figs 19 and 20 for two sets of controller gains. Figures 19(a) and (b) show the behaviour of the PD controller when the proportional gain is equal to 26 and the derivative gain is equal to 9. In Fig. 19(a) and Fig. 20(a) the reference trajectory is the same as simulation (e), and in Fig. 19(b) and Fig. 20(b) the reference signal is same as simulation (c).

Considering Figs 19(a) and (b), although the tuned controller gains result in a stable behaviour for tracking a sinusoidal reference trajectory, the response to the step reference (shown in Fig. 19(b)) is not stable.

The second PD controller is adjusted to ensure an acceptable behaviour for the step reference. In this controller the proportional gain is 8 and the derivative gain is 1. The results are shown in Figs 20(a) and 20(b).

The second tuned PD controller has a roughly acceptable response to the step reference (with very slow convergence rate); however, the tracking performance, shown in Fig. 20(a), suffers from a significant tracking error.

Considering the above results along with Figs 15 and 12, it is obvious that the proposed composite controller has a superior performance and is more reliable behaviour in comparison with simple PD controller. Note that, even when the PD controller results in a stable response (Figs 19(a), 20(a), and 20(b)) the achieved performance is more inferior than the proposed composite technique.

These results exhibit the effectiveness of the proposed technique in comparison with simple (PD) or high level (PFL) classic controllers.

5 EXPERIMENTAL RESULTS

For further validation of the proposed technique and to demonstrate the practical effectiveness of the proposed technique over the conventional methods, a set of experiments is performed. And the results are given in this section. The experimental set-up is a single link flexible manipulator from Quanser [30]. The arm deflection of this set-up is measured via a
strain gauge mounted at the clamped end of the flexible arm. The output of this strain gauge is an analog signal proportional to the deflection of the link. The installation and calibration of this gauge is done via the producing company. So the deflection-output is available in the specific software. Note that, considering equation (4), the tip position can be easily calculated utilizing the measured hub angle in addition to the link deflection. The detailed information upon the utilized flexible link manipulator can be found in reference [30]. The sampling rate is 1msec and the control methodology is implemented in MATLAB. Figure 21 exhibits the utilized experimental set-up.

In this experiment, first the reflected tip output redefinition technique is utilized ($a = -1$) as a well-known conventional technique [11]. The reference input is generated by switching between three diverse sinusoidal signals. In order to evaluate the performance of the system in different frequencies, the aforementioned sinusoidal signals have different frequencies including 5 rad/s, 7 rad/s, and 10 rad/s. The result is given in Fig. 22.

Considering Fig. 22, increasing the frequency content of the reference signal, results in high amplitude undesirable tracking error.

In the next step the outer modifier ($\mu$ controller) is applied. Utilizing the D–K iteration method, the controller is designed [29]. The achieved upper bound of $\mu$ is 0.58. The order of the synthesized controller is 12. Utilizing the Hankel singular value based model reduction technique [31, 32] the order is reduced to 6. The result is given in Fig. 23. The reference trajectory is the same as Fig. 22.

As it can be seen from Figs 22 and 23, the superior performance of the proposed technique utilizing the outer modifier is demonstrated and the tracking error is considerably reduced.

6 CONCLUSIONS

In this paper a new control structure has been proposed for precise tip tracking of flexible link manipulators in uncertain situations. The proposed structure has two major parts: the inner control loop which is a LRFL and the outer control loop which is a robust $\mu$ synthesis based trajectory modifier. Unlike previous research on the flexible link manipulators, in this structure the outer loop is designed in order to modify (online) the reference trajectory for minimizing the tip tracking error. The modified trajectory is the reference signal of the inner controller which is performed on the conventional redefined output. In conventional techniques, redefined outputs cause undesirable oscillations in the tip response. These oscillations have been robustly alleviated in the proposed structure by outer modifier loop. Hence, to have small tip tracking error it is not vital to select the redefined output very close to the tip, and it can be selected inside a safe margin to ensure the stability of the zero dynamics, especially when the payload is uncertain. Finally simulation and experimental results are presented to illustrate the considerable improvements of the performance over the conventional PFL techniques.

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REFERENCES


Consequently an upper bound on $\rho$ can be calculated as (39)

$$
\rho^* \leq \frac{|M_1|_{max}}{1 - |M_2|_{max}} + \frac{|M_2|_{max} - |\bar{y}_r - K_1 \hat{e} - K_2 \hat{e}|}{1 - \frac{B^T \hat{e} \rho}{e} |M_2|_{max}}
$$

Case 2

If $B^T \hat{e} \rho \leq \epsilon$ then:

$$
\rho^* = \left\{ \begin{array}{ll}
\rho_1^* & \text{if } B^T \hat{e} \rho > \epsilon \\
\rho_2^* & \text{if } B^T \hat{e} \rho \leq \epsilon 
\end{array} \right.
$$

It should be noted that, if $\rho^*$ is utilized as an upper bound on $\rho$ then the lower bound on $B$ ($B_{min}$) should satisfy (42)

$$
\left\{ \begin{array}{l}
1. \quad \text{if } B_{min} > B \left( B^T \hat{e} \right)^{-1} > 0 \\
2. \quad \text{sign}(B_{min}) \text{ is equal to sign}(B)
\end{array} \right.
$$

In (39) and (40), $|M_1|_{max}$ and $|M_2|_{max}$ can be calculated as follows. First, considering (33), it is clear that $|M_1| = A - B\hat{B}^{-1} \hat{A}$ and $|M_2| = \hat{B} \hat{B}^{-1}$ so it can be inferred that

$$
\left\{ \begin{array}{l}
\text{if } |A_{max}| \geq |A_{min}| \text{ then:} \\
\quad \text{define } A_1 = A_{max} & \text{and } B_1 = \left( B_{min} \hat{B}^{-1} \right)^{-1} \\
\text{if } |A_{max}| < |A_{min}| \text{ then:} \\
\quad \text{define } A_1 = A_{min} & \text{and } B_1 = \left( B_{max} \hat{B}^{-1} \right)^{-1}
\end{array} \right.
$$

then $|M_1|_{max} = |A_1 - B_1|$ (44)

$$
\left\{ \begin{array}{l}
\text{if } B_{max} \hat{B}^{-1} \geq B_{min} \hat{B}^{-1} \text{ then:} \\
\quad \text{define } B_2 = B_{max} \hat{B}^{-1} \\
\text{if } B_{max} \hat{B}^{-1} < B_{min} \hat{B}^{-1} \text{ then:} \\
\quad \text{define } B_2 = B_{min} \hat{B}^{-1}
\end{array} \right.
$$

then $|M_2|_{max} = |B_2 - B_1|$ (46)

In (43) and (45) $A_{max}$ and $B_{max}$ are the upper bounds on $A$, $B$ respectively. The calculation of $|A_{max}|$, $|B_{max}|$ can be done by knowledge upon $M$, $h_1$, $h_2$, $f_1$, $f_2$, $F_1$, $F_2$, $K$, $f_c$ and their bounds.