Sagittal Optimal Gait of Biped Robot During Double Support Phase (DSP)

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Abstract:
This paper deals with dynamic optimization of bipedal locomotion. The problem of generating optimal sagittal reference gait in bipedal walking is addressed. The main focus of our research is motion optimization of double support phase (DSP). The optimization problem is dealt by using Pontryagin’s Maximum Principle (PMP). For motion optimization of DSP we have considered the closed kinematic chain to be opened at appropriate joint and apply the components of ground reaction forces on the tip of front leg and finally using penalty method to tighten the leg to its prescribed location. The feasible sets of motion are taken into consideration by using inequality constraint to limit the joint motion. The components of ground reaction forces on front leg would also be introduced as control variables in optimization of DSP. The technique we proposed has the ability to generate optimal free motions without specifying joint trajectories and simply by minimizing performance criterion based on joint actuating torques. A final two point boundary value problem is solved by implementing a shooting method. This technique allows for specifying very few parameters to characterized gait pattern, the optimization process has the ability to generate a motion with a minimum of postural and kinematic constraints.

Keywords: Sagittal Gait, Double Support Phase, Bipedal Robot, Dynamics of Walking, Motion Optimization.

1. INTRODUCTION

In recent years the attention of researchers for legged locomotion has continuously been increased in both areas of robotics and biomechanics. The legged robots adopt themselves easily to varying types of grounds, especially the biped robots. They have ability to move themselves in environments defined by a great number of constraints. But, bipedal walking reveals many challenging issues. Apart the numerous technical problems encountered in designing a legged locomotion system with many powered joints, there is a problem of mastering the dynamics of a multi-body systems with sophisticated kinematics and many degrees of freedom. This problem to which the present works devoted has aroused the interest of a number of researchers.

The biped with the simplest kinematics was designed by McGeer [3] as a compass link structure which is able to perform a purely sagittal gait. Dynamic behavior of such a simple system, however, is not obvious. McGeer established that this biped can walk adown a slope by gravity-induced passive motion. Following McGeer, Fomalsky [11] has modeled the dynamics of five-link sagittal biped in order to design impulsive control when considering the double-support phase as instantaneous. In doing so, energy consumption is lowered. However, the impact effect due to impulsive motion control was not considered.

Another interesting technique for generating walking motion of a bipedal robot was presented by Channon et al. [7] they have developed a gait optimization method based on the representation of joint motions by polynomials whose coefficients are adjusted at best in order to minimize the energy consumption. Shin et al. [10] also have carried out trajectory synthesis during the single-support phase of a seven-link, twelve degrees of freedom anthropomorphic robot, by defining a set of pattern parameters that included the specification of kinematic transfer conditions. The techniques shown in [4], [6] and [10] are based on analoguous approach found in the choice of motion kinematic specifications. Goswami and Roussel [12] implemented piecewise constant inputs method for generation of energy optimal complete gait cycle.

In this paper we are interested in achieving purely dynamic synthesis of walk motion of a planar biped during the both single-support phase (cf. [4], [7], [10]) and double-support phase (cf. [12]) without impact at the end of the swing phase.(cf. [1], [6])

This approach allows for a fully dynamic model of the biped, and is based on minimizing the integral of quadric joint actuating torques. Optimal motion synthesis is achieved by applying the Pontryagin’s Maximum Principal. The single support phase can be modeled as an open kinematic chain, but, when the tips of this open kinematic chain is brought to contact with a supporting floor, a closed-loop as a multi-support phase will be resulted and will make the differential equations more complicated.

For modeling the dynamic of such multi-body systems, the closed-loop can be considered as open at constrained joints. Such conditions can be considered as holonomic constraints used for formulating dynamic model with
Lagrangian multiplies. In this way, one can obtains a set of equations (a set of differential algebraic equations) which are not suitable for dealing with dynamic optimization.

The presented approach is based on applying penalty method which releases optimization problem from Lagrangian multipliers. In this method, we consider all closed loops of the multibody system as open at appropriate joints. The constrains then expressing the closure condition are used in the performance criterion of the optimization problem in order to be reduced in the optimization criteria to a minimum value. This in turn will reduce the numerical value of the constrained mentioned earlier.

At the end of this paper we have presented a typical result for a complete gait cycle simulated by this method.

2. KINEMATICS AND DYNAMICS OF THE BIPED

2.1. Kinematic model

We describe a sagittal model of a 5-DOF anthropomorphic biped shown in figure 1. This model is made up of 5 links numbered from $L_1$ to $L_5$. The mass of each link is defined by $m_i$ and $I_i^z$ represents the moment of inertia with respect to the joint axis $O_i$. Such a planar system comprises two ankles, two knees and two coaxial hip joints. The biped motion can be considered by the five relative joint coordinates adapted to generalized coordinates. The joint coordinates and joint velocities can be noted as:

$$\dot{q} = [\dot{q}_1, \ldots, \dot{q}_n]$$

$$q = [q_1, \ldots, q_n]$$

in which $n = 5$.

![Figure 1-Biped kinematic model](image)

2.2. Dynamic model

For formulating the biped dynamics in the Double-Support Phase (DSP), first the set of constrains are assumed to be holonomic. The biped leg during the DSP must be fixed at its prescribed location. The geometrical constrains can be written with respect to generalized coordinates $q_i$.

$$\phi(q) = \left[ \begin{array}{c} \phi_1(q) \\ \phi_2(q) \end{array} \right] = 0 \quad \phi(q) \in \mathbb{R}^m, \ m = 2$$

By using Lagrange's formulation, the dynamic equations of moving constrained body can be written in accordance to applying Lagrangian multipliers.

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i^a + Q_i^d + J_i^T \lambda \quad i = 1, \ldots, n$$

Where $Q_i^a$ (resp. $Q_i^d$) represents the joint actuating torques (resp. joint dissipative torque) exerted by $L_{i-1}$ on $L_i$ at $O_i$ (fig. 1) and $J_i^T$ represents the Jacobean matrix. $\lambda$ represents the forces of constraint which are vertical and horizontal ground reaction forces.

As we have implemented Pontryagin’s Maximum Principal for motion optimization, the main problem in formulating dynamic model of DSP in comparison with single support phase (SSP) is that, we cope with Lagrangian multipliers which are not suitable to be used in PMP. Our approach to overcome this difficulty is to consider closed kinematic chain of the biped to be opened at joint $O_5$, and apply the components of ground reaction forces on the tip of front leg (fig. 2) and finally using the penalty method to tighten the tip of the leg on its prescribed location. In this method the components of reaction forces would be considered as control variables same as actuating torques.

Using penalty techniques holds the fact that firstly the Lagrangian multipliers should be replaced by reaction forces in equation (2) and secondly motion optimization should be done with respect to geometrical constrains which minimizes $\phi(q)$.

![Figure 2-considering closed kinematic chain of biped to be opened in DSP](image)
By these assumptions the equation (2) can be written as:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = Q_i^a + Q_i^d + J_q^T F$$  

(3)

Where \( F \) stands for the ground reaction forces. Let us underline that it is computationally quite efficient to formulate a dynamic model adapted at best to the chosen optimization technique. As we intend to use PMP for solving the dynamic optimization problem, let us recall that the implementation of the PMP requires the formulation of the dynamic model in the state space form. As described in [1] the Hamiltonian dynamic model not only best fulfills the requirement, but also strengthens the robustness of the algorithms used to solve the optimization. Now we present the outlines of the formulation we need.

Defining the conjugate momenta and Hamiltonian as:

$$p_i = \frac{\partial L}{\partial \dot{q}_i}, \quad i = 1,...,n$$  

(4)

$$H(q,p) = p^T \dot{q} - L(q,\dot{q})$$

Lagrange's equations in (3) may be formulated in Hamiltonian form:

$$\dot{q}_i = \frac{\partial H}{\partial p_i}$$

$$\dot{p}_i = -\frac{\partial H}{\partial q_i} + Q_i^a + Q_i^d + J_q^T F$$  

(5)

The expression of \( p \) can be written through (4) as 

$$p = Aq$$

in which \( A \) being the \((n \times n)\) mass matrix of the kinematic chain. Then equations (3) become more explicitly:

$$\dot{q}_i = \sum_{j=1}^{n} A_{ij}^{-1} p_j$$

$$\dot{p}_i = -\frac{1}{2} p^T A_{ii}^{-1} p - V_{,i} + Q_i^a + Q_i^d + J_q^T F$$  

(6)

where \( V \) stands for the gravity potential and:

$$A_{ii}^{-1} = \frac{\partial A^{-1}}{\partial q_i}, \quad V_{,i} = \frac{\partial V}{\partial q_i}$$  

(7)

With this formulation, Hamiltonian equations are ideally structured for applying the Pontryagin's Maximum Principle. Now defining the state variables and control variables as:

$$x = (x_1,...,x_{2n})^T \equiv (q_1,...,q_n,p_1,...,p_n)^T$$  

(8)

$$u = (u_1,...,u_{n+m})^T \equiv (Q_1^a,...,Q_n^a, F_n, F_r)^T$$  

(9)

where \( u_1 \) through \( u_n \) represent joint actuating torques and \( F_n \) (resp. \( F_r \)) represents the normal (resp. horizontal) component of ground reaction force. (Fig 2)

The double set of vectorial equations (6) can be reacts as the 2n-order differential vector-equation:

$$\ddot{x}(t) = F(x(t)) + B(x(t))u(t)$$  

(10)

In above equation, initial and final states will be specified as :

$$x(t') = x', \quad x(t'') = x''$$  

(11)

3. FEASIBLE MOTIONS AND CONSTRAINTS

Feasible motions of the biped are defined by two types of specific conditions. The first type consists of limiting the joint actuating torques. The second type specifies interaction conditions between the stances foot and the ground.

3.1. Box constraints on joint actuating torques

Torques produced by actuators have limited values. When they are considered at the joint level, we can write:

$$\forall t \in [t', t''], \quad |Q_i^a(t)| \leq Q_i^{a,\text{max}}$$  

(12)

3.2. Unilaterality of contact

The unilaterality of contact expressed by the fact that the vertical component of ground reaction forces must remain positive during the motion. This condition simply means that the foot is not stuck on the ground and that the ground cannot pull but only push it.

With this expression the unilaterality condition are expressed by :

$$\forall t \in [t', t''], \quad 0 < F_n^\text{min} \leq F_n(t)$$  

(13)

The latter condition can be expressed by non-sliding of the foot on the ground.

$$\forall t \in [t', t''], \quad |F_i(t)| \leq \mu F_n(t)$$  

(14)

Equations (12), (13) and (14) define the space of control variables \( U \).

4. FORMULATING AN OPTIMAL CONTROL PROBLEM

We wish to generate an optimal motion in complete step by minimizing a performance criterion representing a dynamic cost. In optimization problem, we have the choice between minimizing actuating torques, or energy
expenditure. Since the biped stands and moves in a vertical plane, it is submitted to the gravity. For this reason, we have favored the first choice by introducing the integral cost

$$J(u) = \int_0^T L(x(t), u(t)) \, dt$$

(15)

where the Lagrangian is the quadratic function of the normalized control variables \( u_i \)

$$L(x, u) = \frac{1}{2} \sum_{i=1}^{n+m} \xi_i (u_i / u_i^{ref})^2$$

(16)

Where \( \xi_i \) are weighting factors and \( u_i / u_i^{ref} \) represent dimensional joint actuating torques and dimension ground reaction forces. The reference value of the reaction forces can be chosen as biped weight. Let us mention that the weighting coefficient can play a much more useful role in the problem we are stating. When \( \xi_i \) increases, the optimal corresponding \( u_i \) is lowered. We will make use of this possibility to reduce the action of actuating torques and master directly the ground reaction forces applied to the tip of front leg.

4.1. Dealing with the geometrical constraints in DSP

Such geometrical constraints defined in equation (1) can be easily dealt with using computing techniques similar to penalty method developed in the frame of mathematical programming. The penalty method consists in minimizing the geometrical constraints functions. In this method the geometrical constraints must be added to optimization criteria as a quadratic term.

$$J_r(u) = J(u) + \frac{r}{2} \| \phi(x) \|^2 \, dt \quad r > 0$$

(17)

The function \( J_r \) must be minimized with sufficiently great value of the penalty multiplier \( r \).

4.2. Applying Pontryagin’s Maximum Principle

The minimization problem may be summarized as : find a phase trajectory \( t \rightarrow x(t) \) and a control vector \( t \rightarrow u(t) \) minimizing \( J_r \), namely

$$\min_{u \in U} J_r(u) \quad r \text{ great}$$

(18)

and satisfying the state equations (10) together with end conditions (11). At this point we define the Pontryagin’s function as:

$$H(x, u, w) = w^T (F(x) + B(x)u) - L_r(x, u)$$

(19)

The maximum principle [9] states that if \( t \rightarrow (x(t), u(t)) \) is a solution of (18-19) with state equations (10-11), then there exists a costate function \( t \rightarrow w(t), w \in \mathbb{R}^{2n} \), satisfying the costate equation:

$$\dot{w}(t)^T = - \frac{\partial H}{\partial x}$$

(20)

and maximality condition

$$H(x, u, w) = \max H(x, v, w) \quad v \in U$$

(21)

A prominent interest of the PMP lies in condition (21) which allows the constraint on \( u(t) \) to be exactly satisfied, and yields trough (10), (16) and (19) an explicit expression of the optimal control variables [1].

The unknown functions \( x \) and \( w \) appear as a solution of a 4n-order differential system of the type:

$$t \in [t', t''] \wedge \begin{cases} \dot{x}(t) = F_1(x(t), w(t)) \\ \dot{w}(t) = F_2(x(t), w(t)) \end{cases}$$

(22)

accompanied by the boundary conditions (11). Typically, we are in the presence of a two-point boundary value problem.

5. NUMERICAL SIMULATIONS

The two point boundary value problem (22) can be solved using computing techniques such as finite difference algorithms or shooting methods. We have chosen the latter approach for its efficiency, and simplicity of its implementation. The technique we use is described in reference [8] as the so-called transition matrix method. Due to the strong non-linearity of dynamic equations, the main difficulty to overcome in order for algorithms to converge toward an optimal solution consists in finding a sufficiently accurate guess. We have used first-order gradient algorithms to overcome this difficulty [8].

According to figure 1 the biped model has the following specification:

<table>
<thead>
<tr>
<th>Table 1 Dimensional characteristics of the biped</th>
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<tbody>
<tr>
<td>Link 1</td>
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<tr>
<td>--------</td>
</tr>
<tr>
<td>Mass(kg)</td>
</tr>
<tr>
<td>Length(m)</td>
</tr>
<tr>
<td>C.O.G</td>
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<tr>
<td>( I_z )</td>
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</tbody>
</table>

The technique we presented in this study allows for simulation of both single-support and double-support
phase. The main difference in double-support phase in comparison with single support phase is that, we have considered the closed kinematic chain to be opened and have applied the components of ground reaction forces on tip of the leg the consequently in this phase penalty factor has a great numerical value, but in single-support phase this factor is zero.

The typical simulated optimal motion has been shown in figure 3. The step length is equal to 0.40(m) and total motion time is 0.43 (s). The average horizontal hip velocity in SSP is equal to 0.95 (m/s) and in DSP is equal to 1.0 (m/s), which is equal to average speed of human gait. We have ignored the impact phase at the end of SSP same as [1].

![Figure 3- Optimal motion of the Biped during a complete gait cycle for step length of 0.4 m](image)

![Figure 4- joint relative velocities](image)

![Figure 5-time variation of actuating torques](image)

![Figure 6-time variation of ground reaction forces](image)

Figure 4 and 5 show the variation of joint relative velocities and actuating torques. The ankle actuating torque of stance leg has been saturated at the end of double-support phase. Introducing a sufficiently great value of the weighting factor $\xi_i$ in formula (16) weakens

the ankle torque of stance leg during DSP.

$$E = \int_0^T \sum_{i=1}^n |\dot{q}_i(t)Q_i^e(t)|dt$$

(23)

The energy expenditure during single support phase is equal to 200.2 J. and during the double support phase is equal to 23.5 J.

6. CONCLUSION

We examined the optimal motion of a 5-DOF biped robot during a complete gait cycle. We implemented Pontryagin's maximum principal for motion optimization of both SSP and DSP. The closed kinematic chain in DSP
phase. The main difference in double-support phase in comparison with single support phase is that, we have considered the closed kinematic chain to be opened and have applied the components of ground reaction forces on tip of the leg the consequently in this phase penalty factor has a great numerical value, but in single- support phase this factor is zero.

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![Figure 5-time variation of actuating torques](image)

Figure 5-time variation of actuating torques

The ankle torque of stance leg during DSP.

![Figure 6-time variation of ground reaction forces](image)

Figure 6-time variation of ground reaction forces

Figure 6 shows the components of ground reaction forces on both legs during both phases. One can mention that in DSP the piped weight has been slightly transferred to front leg. As the components of reaction forces on front leg in DSP have been considered as control variables, this method simply allows mastering directly theses components.

Energy consumption is computed using the formula

$$E = \int \sum_{i=1}^{n} \dot{q}_i(t) Q_i^a(t) dt$$  \hspace{1cm} (23)

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We examined the optimal motion of a 5-DOF biped robot during a complete gait cycle. We implemented Pontryagin’s maximum principal for motion optimization of both SSP and DSP. The closed kinematic chain in DSP
considered to be opened and geometrical constraints dealt by the mean of penalty technique. The technique we presented allows for generating smooth motions with minimum kinematical constraints and to master directly the control variables as interaction forces.

To conclude, the future work would consist in matching optimally both phases of gait.

References