Fuzzy Model Predictive Control Applied to Piecewise Linear Systems

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Abstract

This paper presents the application of a combined control strategy applied to nonlinear systems and grounded in a MPC structure using a fuzzy model description. The nonlinear models were treated as several sub-models with linear behavior, called Piecewise Linear systems (PWL). The advantages of this methodology were visualized in the temperature control of a continuous tank reactor with output multiplicity behavior. Comparisons with classical control approaches showed that the PWL MPC is an attractive and practical strategy.

Keywords: fuzzy control, MPC, piecewise linear systems (PWL)

1. Introduction

Model Predictive Control (MPC) has called a great deal of attention in the industry, and today is the controller of choice for many areas of chemical and petrochemical industry. The great advantage of the linear model based predictive control (LMBPC) approach consists in solving a convex cost function instead of a non convex one, which has no guarantees to be solved in time. However, due to the intrinsic nonlinearity of chemical industries, obtaining good first-principles model of a process is a non-trivial task. Furthermore, an input-output data based identified model has its quality severely reduced when it is moved away from its designed operating point. An efficient alternative is the utilization of fuzzy modeling that can process numerical or language information and thus have the possibility of inclusion of qualitative information in the description of the process plant to be controlled.

Literature on fuzzy models (Takagi & Sugeno, 1985) for model predictive control covers both the utilization of fuzzy black-box models (Jang, 1993; Babuska et al., 1998) and the approximation of nonlinearities characteristics using multiple linear models. Fischer et al. (1997) and Espinosa et al. (1998) investigated the utilization of an identified fuzzy model in parallel with the MPC structure which generates step response coefficients at every sampling time. Roubos et al. (1999) showed the utilization of TS models as linear models with state dependent parameters. Huang et al. (2000) reported the approximation of nonlinear models using multiple step response convolution models; Marusak (2007), in a similar but more efficient way, designed a fuzzy model predictive controller applied to a non-minimum phase reaction system.

This paper aims to design and investigate the predictive control based on fuzzy logic models. The multiple step response approach presented by Marusak (2007) is examined...
and discussed; it is known that the great drawback of this formulation is the over
parameterized models and its restriction to describe only asymptotically stable plants.
As an alternative for this problem, a piecewise linear state space model approach is
proposed. Although less intuitive than the step response models, state space models are
more informative, can easily treat multiple input – multiple output (MIMO) problems
and are well suited for non-stable plants as well. The strategy and the developed
controllers are illustrated in the control of a highly nonlinear continuous reactor. Results
and comparisons with classical techniques of process control are addressed in the study
of the MPC introduced in this paper.

2. Piecewise Linear Model Predictive Control

In the basic structure of MPC, both the model and optimizer act together in order to
handle future and past inputs and outputs, cost function and reference trajectory
generating the future errors and dealing with constraints. In order to compute de optimal
control signal, an objective function (1) is minimized, subject to constraints in the input,
input speed, and output.

\[
J = \sum_{k=H_w}^{H_p} \left\| \hat{y}(t+k) - w(t+k) \right\|^2_Q + \sum_{k=0}^{H_c-1} \left\| \Delta \hat{u} \right\|^2_R
\]

(1)

Here \(H_w\) is the process delay, \(H_p\) is the predicted horizon, \(H_c\) is the control horizon, \(\hat{y}\) is
the output of the plant model, \(w\) is the reference trajectory and \(\Delta \hat{u}\) is the future control
increment. For a control horizon \(H_c > 1\), a vector of inputs are returned by the
optimization, but only the first term is really sent to the process. For a detailed revision
of MPC, see Maciejowski (2002).

Let us assume a nonlinear process represented by (2), where \(x\) is the state vector and
\(u\) is the input of the process.

\[
\frac{dx}{dt} = f(x,u) \quad \text{and} \quad y = g(x,u)
\]

(2)

A linear model could only represent the dynamic characteristics of the process in the
neighborhood of a specified equilibrium point; restricting the use of the predictive
controller to the boundaries of the linear model. As an alternative, the nonlinear model
can be described by several linear sub-models known as piecewise linear models
(PWL), which in conjunction represents the whole dynamic behavior of the nonlinear
system. The PWL methodology allows the switching over the entire representation of
the process (see Figure 2.1), minimizing the prediction quality losses caused by
movements over the operational trajectory.

Figure 2.1 Transition of piecewise linear models.
These transitions can be adjusted by higher order TS models, with rules representing each linear region of an overall nonlinear model as an LTV (linear time variant) system. In this case, the estimated output is either a mean value between the two models without intersection (R3 and R4) or the weighted model response of two close operational points (R1 and R2).

2.1. Takagi-Sugeno Fuzzy Models

Takagi-Sugeno fuzzy models can be described for a multiple input single output (MISO) system as a set of \(i = (1, 2, 3... l)\) rules in the following form:

\[
R_i: \text{IF } u_1 \text{ is } A_1 \text{ AND } u_2 \text{ is } A_2 \text{ AND} \ldots \text{AND } u_n \text{ is } A_n \text{ THEN } \hat{y}_i(k) = f(u_1, u_2, \ldots, u_n) \tag{3}
\]

where \(u_1 \ldots u_n\) are inputs of the fuzzy model, \(A_1 \ldots A_n\) are fuzzy sets represented by membership functions which weights a crisp input \(u_n\) in a degree of fulfilment such as \(w_{jn}(u_n): \mathbb{R} \rightarrow [0,1]\); \(\hat{y}_i\) is the linear model of the \(i^{th}\) rule. The resultant model (4) is given by a \(t\)-norm operation of the inputs and the normalized weights (5) in the composition of the \(l\) rules.

\[
\hat{y} = \Phi_1 y_1 + \Phi_2 y_2 + \ldots + \Phi_l y_l \tag{4}
\]

\[
\Phi_i = \frac{w_i}{\sum_{j=1}^{l} w_j}; \quad \sum_{i=1}^{l} \Phi_i = 1 \tag{5}
\]

For a piecewise step response model (6) with a horizon \(H\), the weighting function \(\Phi\) is applied by means of superposition principle over the step response coefficients of each linear model.

\[
\hat{y}(t+k) = \sum_{m=k+1}^{H-1} s^*_m \Delta u(t+k-m) + s^*_r u(t+k-H); \quad s^*_j = \sum_{j=1}^{l} \Phi_j s_j \tag{6}
\]

Here \(s^*\) is the weighted toeplitz matrix of the step response coefficients and \(s_j\) is the matrix representing the \(j^{th}\) sub-model of the PWL or PWA (Piecewise Affine) system.

The PWA representation of a discrete state space model (7) is similar to the above step response model. The state matrix \(A^*\) and the input matrix \(B^*\) are also weighted by the function \(\Phi\), while \(A_j\) and \(B_j\) matrix (8) represents the local linearization at the \(j^{th}\) equilibrium point of a nonlinear model.

\[
\begin{align*}
\hat{x}(k+1) &= A^* \hat{x}(k) + B^*u(k) \\
\hat{y}(k) &= C^* \hat{x}(k) + D^*u(k)
\end{align*} \tag{7}
\]

\[
A^* = \sum_{j=1}^{l} \Phi_j A_j; \quad B^* = \sum_{j=1}^{l} \Phi_j B_j; \quad C^* = \sum_{j=1}^{l} \Phi_j C_j; \quad D^* = \sum_{j=1}^{l} \Phi_j D_j \tag{8}
\]

In Figure 2.2, a simple representation of the PWL Model Predictive control is placed in the sense of an internal model structure (Roubos et al., 1999). The bias correction \((\hat{y}(t) - \hat{y}(t))\) is used to prevent modeling errors and in the estimation of unmeasured disturbances. The sub-models are composed by means of fuzzy reasoning to a single
model. The predicted output vector is sent to the optimizer and the quadratic convex function is solved for $\Delta u(t)$.

![Diagram of PWL Model Predictive control scheme](image)

Figure 2.2 PWL Model Predictive control scheme.

3. Control Problem

A non-isothermal Continuous Stirred Tank Reactor (CSTR) was utilized as a control benchmark problem (see Figure 3.1a) for the PWL MPC. The nonlinear system presented by Luyben (1995) consists of an irreversible, exothermic reaction ($A \rightarrow B$) carried out in a perfectly mixed CSTR. The three state model is given by the reactor temperature, $T$ (K), the reactor feed concentration, $C_A$ (kmol.m$^{-3}$), and the jacket reactor temperature $T_j$(K). The problem consists in controlling the reactor temperature manipulating the makeup jacket flow, $F_j$(m$^3$/hr), bringing the process to a desirable product concentration, $C_B$ (kmol.m$^{-3}$), value.

The motivation in using this example is due to the characteristics it presents with the output multiplicity behavior of the system. Here, a single input can lead the process to three different output values as shown in Figure 3.1b. For this system, the intermediate region is unstable in open-loop. For the open-loop case, depending on the input signal, the system can go to an ignition temperature (higher temperature region) or to an extinction temperature (lower temperature region) (Bequete, 2007).

![Figure 3.1 (a) Representation of the Continuous Stirred Tank Reactor (CSTR) problem; (b) Multiple steady-state behavior for the proposed control problem.](image)
4. Results

Two PWL MPC controllers were designed for the temperature control of the proposed problem starting with $F_{ss}=1.41\text{m}^3/\text{hr}$ at a sampling time of $0.08\text{hr}$; A dynamic matrix controller (DMC) was developed for the stable low temperature region ($C_{ss}=7.6\text{kmol/m}^3$, $T_{ss}=298.42\text{K}$) and a state space model predictive controller (SSMPC) for the intermediate unstable region ($C_{ss}=3.92\text{kmol/m}^3$, $T_{ss}=333.34\text{K}$). For the comparisons, a standard MPC of each type and a PI (proportional-integral) controller optimally tuned for the reference trajectory were utilized.

For the standard DMC algorithm, a single step response is taken in the region $R2$. The PWL DMC takes two additional regions into account covering a large area of operation given by $R1$ and $R3$. For the SSMPC the model $Q1$ is taken by linearization of the nonlinear model in the limits of $T=333.05\text{K}$. The PWL SSMPC takes an additional model $Q2$ in the limits of the temperature $T=335.83\text{K}$. The fuzzy sets for the PWL controllers were chosen heuristically and are given in Figure 4.1.

In both simulations, the system was submitted to setpoint changes making the reactor temperature pass along the regions mentioned above. Control movements were penalized by a weighing factor $R=0.01$ for both configurations of MPC. SSMPC was subject to constraints in the changing of control signal ($\Delta u\leq0.425\text{m}^3/\text{hr}$) preventing large temperature overshoots.

![Figure 4.1](image1.png)  
Figure 4.1 (a) Fuzzy sets for the PWL DMC; (b) Fuzzy sets for the PWL SSMPC.

![Figure 4.2](image2.png)  
Figure 4.2 Closed-loop behavior of reactor temperature: (a) PWL DMC controller ($H_c=2$, $H_p=4$); (b) PWL SSMPC controller ($H_c=10$, $H_p=15$).
The closed-loop behavior of the system is given in Figure 4.2. Results showed that, for the lower temperature region (see Figure 4.2a), the PWL DMC had a better response due to the correction provided by the multiple sub-models approach on the final gain, leading the controller to a faster response. In Figure 4.3b the PWL SSMPC controller showed a slightly better response, the similar behavior were due to the high interpolation between the two models $Q_1$ and $Q_2$. A small interpolation of the models was found to be unsatisfactory, due to the high nonlinearity and instability of that region. Furthermore, the great advantage of the proposed approach is the possibility to walk in the entire trajectory of the intermediate region, which cannot be done with a single model, depending on the nature of the operating point. It is shown that the PI controller is able to perform reasonably well, at the great cost of control moves and overshoot.

5. Conclusions
The PWL MPC based on a fuzzy logic description is introduced and illustrated with success in nonlinear process with output multiplicity. The overall performance of the controller depends not only on the number of local models considered, but also how the membership function is defined. Perhaps complex systems will require an optimization procedure for designing the membership region for each model. The structure presented herein is valid and can be easily adapted to any linear MPC strategy. The controller can be easily adapted to several cost function norms and can incorporate robustness constraints with easy.

References
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