

054414 Process Control System Design

LECTURE 12: MODEL PREDICTIVE CONTROL

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12-1 PROCESS CONTROL SYSTEM DESIGN - (c) Daniel R. Lewin Model Predictive Control

Objectives

On completing this section, you should:

- ❶ Be able to explain to a layman how model predictive control (MPC) works.
- ❷ Understand why MPC can improve on the performance that can be expected from decentralized controllers.
- ❸ Be able to correctly tune MPC to obtain a good response.
- ❹ Be able to implement MPC using MATLAB.

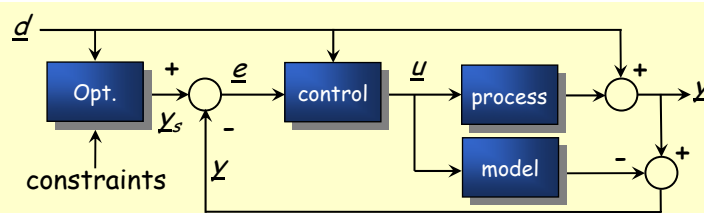
Refs: 1) Seborg, D. E., Edgar, T. F. and Mellichamp, D. A., Process Dynamics and Control, 2nd Ed., Wiley (2004).
2) Ogunnaike, B. A. and Ray, W. H., Process Dynamics, Modeling and Control, Oxford, pp. 1000-1008 (1994).

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Introduction

Thus far, digital controllers have been designed using relatively simple models to predict the future values of the process outputs. We shall now consider a more general form of model predictive control in which:

- ❶ The set points are computed based on a constrained optimization of a steady-state process model.
- ❷ A model-based predictive algorithm ensures that the process outputs track the setpoints over a prespecified horizon.



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Constrained Optimization

The most common approach is to perform constrained optimization using a static process model at regular intervals. This ensures that the process is maintained at the optimal operating level when confronted with such things as:

- Changes in product demands.
- Disturbances such as feedstock changes
- Changes in equipment availability

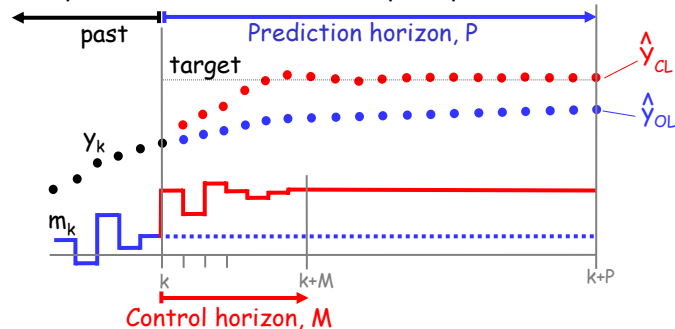
Despite the fact that processes can be highly nonlinear, the most commonly-used method is LP. This has been covered in other courses (054374, 054402), so it will not be discussed further here. This leaves MPC to cover...

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MPC made simple...

MPC is a model-based control strategy using models in 2 ways:

- Uses a reliable **model** to predict effect of past control moves on P future outputs, assuming no future moves.
- Uses the same **model** to compute the optimal M controller moves. Implement first move and repeat procedure.

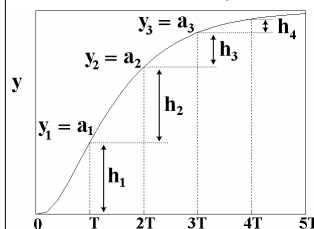


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Discrete Convolution Models

Evidently, a discrete model describing the effect of inputs on outputs is required. Commonly, discrete impulse response, or convolution, models are used.

Consider the response of a typical process to a unit step input:



a_i are the step response coefficients
 h_i are the impulse response coefficients
 $h_i = a_i - a_{i-1}, i = 1, \dots, H$ - model horizon.
 $h_0 = 0$ (what does this assume?)

H , the model horizon, is selected to capture 99% of the transient.

A discrete convolution model is: $\tilde{y}_{k+1} = \sum_{i=1}^H h_i m_{k+1-i}$

Predicted value of y

Manipulated variable at $k+1-i$

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Discrete Convolution Models

Discrete convolution model: $\tilde{y}_{k+1} = \sum_{i=1}^H h_i m_{k+1-i}$ (11.1)

$$\tilde{y}_{k+1} = h_1 m_k + h_2 m_{k-1} + \dots + h_H m_{k+1-H} \quad (11.2)$$

where $m_{k+1-H} = 0$ when $k+1-H < 0$

Define $\Delta m_k = m_k - m_{k-1}$ and substituting in (11.2):

$$\begin{aligned} \tilde{y}_{k+1} &= h_1 m_k + h_2 m_{k-1} + \dots + h_H m_{k+1-H} \\ &= (a_1 - a_0) m_k + (a_2 - a_1) m_{k-1} + \dots + (a_H - a_{H-1}) m_{k+1-H} \\ &= a_0 m_k + a_1 (m_k - m_{k-1}) + \dots + a_{H-1} (m_{k+1-H} - m_{k+1-H}) + a_H m_{k+1-H} \end{aligned}$$

Hence: $\tilde{y}_{k+1} = \sum_{i=1}^H a_i \Delta m_{k+1-i}$ (11.3)

Note that this assumes that $\Delta m_{k+1-H} = m_{k+1-H} - m_{k-H} = m_{n+1-H}$

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Discrete Convolution Models

Applying Eq. (11.3), a step change in m at $k=0 \Rightarrow \Delta m_0 = m_0 - m_{-1}$, with $\Delta m_k = 0, k > 0$:

$$\begin{aligned} k=0: \tilde{y}_1 &= \sum_{i=1}^H a_i \Delta m_{1-i} \\ &= a_1 \Delta m_0 + a_2 \cancel{\Delta m_{-1}} + \dots = a_1 \Delta m_0 \end{aligned}$$

$$\begin{aligned} k=1: \tilde{y}_2 &= \sum_{i=1}^H a_i \Delta m_{2-i} \\ &= a_1 \cancel{\Delta m_1} + a_2 \Delta m_0 + a_3 \cancel{\Delta m_{-1}} + \dots = a_2 \Delta m_0 \end{aligned}$$

In general, $\tilde{y}_k = a_k \Delta m_0$ - predicted values are step response coefficients multiplied by Δm_0 .

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ZT-Convolution Model Equivalence

We shall now see how these convolution models are equivalent to transfer function models studied so far. As an example, consider the process: $p(s) = \frac{1}{s+1}$ with a hold and $T = 0.2$.

$$\text{Then } HP(z) = \frac{(1 - e^{-0.2})z^{-1}}{1 - e^{-0.2}z^{-1}} = \frac{0.1813z^{-1}}{1 - 0.8187z^{-1}}$$

$$\text{By long division: } HP(z) = 0.1813z^{-1} + 0.1484z^{-2} + 0.1215z^{-3} + 0.0995z^{-4} + 0.0815z^{-5} + \dots \quad (11.4)$$

To obtain unit step response: $\hat{Y}(z) = HP(z)M(z) = HP(z) \cdot \frac{1}{1-z^{-1}}$

$$\text{By long division: } \hat{Y}(z) = \underbrace{0.1813z^{-1}}_{y(T)} + \underbrace{0.3297z^{-2}}_{y(2T)} + \underbrace{0.4512z^{-3}}_{y(3T)} + 0.5507z^{-4} + 0.6321z^{-5} + \dots \quad (11.5)$$

This is identical to the response: $y(kT) = 1 - e^{-kT}$

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ZT-Convolution Model Equivalence

Note that Eq. (11.3) is $\tilde{y}_{k+1} = \sum_{i=1}^H a_i \Delta m_{k+1-i}$

Taking Z-transforms of this equation gives:

$$\hat{Y}(z)z^{k+1} = \sum_{i=1}^H a_i \Delta \hat{M}(z) z^{k+1-i} \quad \rightarrow \quad \frac{\hat{Y}(z)}{\Delta \hat{M}(z)} = \sum_{i=1}^H a_i z^{-i} \quad (11.6)$$

Hence, when $\Delta m_0 = 1$ ($\Delta \hat{M}(z)$), Eq. (11.6) is equivalent to Eq. (11.5), so coefficients $a_1 = 0.1813$, $a_2 = 0.3297$, etc., can be obtained by measuring the unit-step response.

Furthermore, we can write Eq. (11.1): $\tilde{y}_{k+1} = \sum_{i=1}^H h_i m_{k+1-i}$

$$\text{in the Z-domain: } \frac{\hat{Y}(z)}{\hat{M}(z)} = HP(z) = \sum_{i=1}^H h_i z^{-i} \quad (11.7)$$

Hence, Eq. (11.7) is equivalent to Eq. (11.4), whose coefficients are h_1, h_2 , etc. and we can check that $h_i = a_i - a_{i-1}$.

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Matrix Form for Predictive Control

In general, at sample k , we will compute:

M future input changes - $\Delta m_k, \Delta m_{k+1}, \dots, \Delta m_{k+M-1}$
 \input horizon

and P predicted future outputs - $\tilde{y}_{k+1}, \tilde{y}_{k+2}, \dots, \tilde{y}_{k+P}$
 \prediction horizon

Generalizing Eq. (11.4):

$$\tilde{y}_{k+1} = \sum_{i=1}^H a_i \Delta m_{k+1-i} = \underbrace{a_1 \Delta m_k}_{\text{future}} + \underbrace{\sum_{i=2}^H a_i \Delta m_{k+1-i}}_{\text{past}}$$

$$\tilde{y}_{k+2} = \sum_{i=1}^H a_i \Delta m_{k+2-i} = \underbrace{\sum_{i=1}^2 a_i \Delta m_{k+2-i}}_{\text{future}} + \underbrace{\sum_{i=3}^H a_i \Delta m_{k+2-i}}_{\text{past}}$$

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Matrix Form for Predictive Control

$$\tilde{y}_{k+M} = \underbrace{\sum_{i=1}^M a_i \Delta m_{k+M-i}}_{\text{future}} + \underbrace{\sum_{i=M+1}^H a_i \Delta m_{k+M-i}}_{\text{past}}$$

$$\tilde{y}_{k+P} = \underbrace{\sum_{i=P-M+1}^P a_i \Delta m_{k+P-i}}_{\text{future}} + \underbrace{\sum_{i=P+1}^H a_i \Delta m_{k+P-i}}_{\text{past}}$$

In summary, at sample interval $n + j$:

$$\tilde{y}_{k+j} = \underbrace{\sum_{i=1}^j a_i \Delta m_{k+j-i}}_{\text{future adjustments}} + \underbrace{\sum_{i=j+1}^H a_i \Delta m_{k+j-i}}_{\text{past adjustments}} \quad j \leq M \quad (11.8)$$

$$\tilde{y}_{k+j} = \underbrace{\sum_{i=j-M+1}^j a_i \Delta m_{k+j-i}}_{\text{future adjustments}} + \underbrace{\sum_{i=j+1}^H a_i \Delta m_{k+j-i}}_{\text{past adjustments}} \quad M < j \leq P$$

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Matrix Form for Predictive Control

Thus, in matrix form, the predicted trajectory is:

$$\begin{bmatrix} \tilde{y}_{k+1} \\ \tilde{y}_{k+2} \\ \tilde{y}_{k+3} \\ \vdots \\ \tilde{y}_{k+M} \\ \vdots \\ \tilde{y}_{k+P} \end{bmatrix} = \begin{bmatrix} a_1 & 0 & 0 & \cdots & 0 \\ a_2 & a_1 & 0 & \cdots & 0 \\ a_3 & a_2 & a_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_M & a_{M-1} & a_{M-2} & \cdots & a_1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_P & a_{P-1} & a_{P-2} & \cdots & a_{P-M+1} \end{bmatrix} \begin{bmatrix} \Delta m_k \\ \Delta m_{k+1} \\ \Delta m_{k+2} \\ \vdots \\ \Delta m_{k+M-1} \end{bmatrix} + \begin{bmatrix} \tilde{y}_{k+1}^0 \\ \tilde{y}_{k+2}^0 \\ \tilde{y}_{k+3}^0 \\ \vdots \\ \tilde{y}_{k+M}^0 \\ \vdots \\ \tilde{y}_{k+P}^0 \end{bmatrix} + \begin{bmatrix} w_k \\ w_{k+1} \\ w_{k+2} \\ \vdots \\ w_{k+M-1} \\ \vdots \\ w_{k+P-1} \end{bmatrix}$$

$\underbrace{\quad}_{\tilde{y}_k} \quad \quad \quad \underbrace{\quad}_{\underline{A}} \quad \quad \quad \underbrace{\quad}_{\Delta \underline{m}_k} \quad \quad \quad \underbrace{\quad}_{\tilde{y}_k^0} \quad \quad \quad \underbrace{\quad}_{\underline{w}_k}$

In the above, \tilde{y}_{k+j}^0 is the output predicted without further control action: $\tilde{y}_{k+j}^0 = \sum_{i=j+1}^H a_i \Delta m_{k+j-i}$ and \underline{w}_k is disturbance effect.

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Dynamic Matrix Control

That, is, in matrix form: $\tilde{y}_k = \underline{A} \cdot \Delta \underline{m}_k + \tilde{y}_k^0 + \underline{w}_k$ (11.9)

In DMC, the effect of disturbances on the output at $k+1$ is considered to be modeling error, and is estimated by using the actual value of y measured at instant k :

$$\hat{w}_k = y_k - \tilde{y}_k \quad (11.10)$$

Now, our objective is to obtain a desired trajectory, \tilde{y}_k^*

Thus, we need to minimize the closed loop prediction error:

$$\underline{e}_k^* = \tilde{y}_k^* - \underline{A} \cdot \Delta \underline{m}_k - \tilde{y}_k^0 - \hat{w}_k$$

Rearranging: $\underbrace{\underline{e}_k^*}_{\text{Closed-loop prediction error}} = \underbrace{-\underline{A} \cdot \Delta \underline{m}_k}_{\text{Vector of future control moves}} + \underbrace{(\tilde{y}_k^* - \tilde{y}_k^0 - \hat{w}_k)}_{\text{Open-loop prediction error}} \quad (11.11)$

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Dynamic Matrix Control

Clearly, we would like to minimize the closed-loop prediction error, \underline{e}_k . This can be formally stated as follows:

$$\min_{\Delta \underline{m}_k} \phi = [\underline{e}_k^0 - \underline{A} \cdot \Delta \underline{m}_k]^T [\underline{e}_k^0 - \underline{A} \cdot \Delta \underline{m}_k] \quad (11.12)$$

where $\underline{e}_k^0 = \underline{y}_k^* - \underline{y}_k^0 - \hat{\underline{w}}_k$ is the open-loop prediction error.

Differentiating Eq. (11.12) with respect to $\Delta \underline{m}_k$, and equating to zero gives:

$$\frac{\partial \phi}{\partial \Delta \underline{m}_k} = -\underline{A}^T [\underline{e}_k^0 - \underline{A} \cdot \Delta \underline{m}_k] = 0$$

$$\text{Solving gives: } \Delta \underline{m}_k = \underbrace{(\underline{A}^T \underline{A})^{-1}}_{\substack{\text{A MxP matrix} \\ \text{of controller} \\ \text{gains}}} \underline{A}^T \underline{e}_k^0 \quad (11.13)$$

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Dynamic Matrix Control

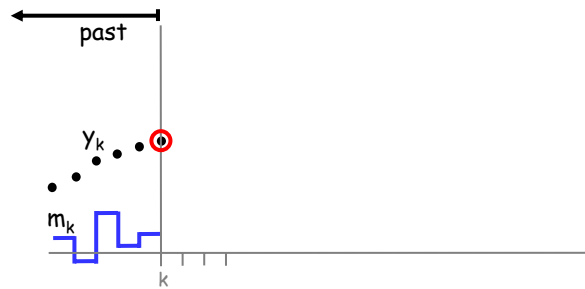
The generic DMC algorithm is implemented as follows:

- ❶ The process output, \underline{y}_k , is measured, and used to estimate the process model error using Eq. (11.10).
- ❷ The open-loop prediction error, $\underline{e}_k^0 = \underline{y}_k^* - \underline{y}_k^0 - \hat{\underline{w}}_k$, is updated (accounting for changes in setpoint and effect of previous controller moves)
- ❸ Solve Eq. (11.13): $\Delta \underline{m}_k = (\underline{A}^T \underline{A})^{-1} \underline{A}^T \underline{e}_k^0$
- ❹ $\Delta \underline{m}_k$ (first step only) is implemented.
- ❺ Counter is updated: $k = k + 1$.

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MPC in Action

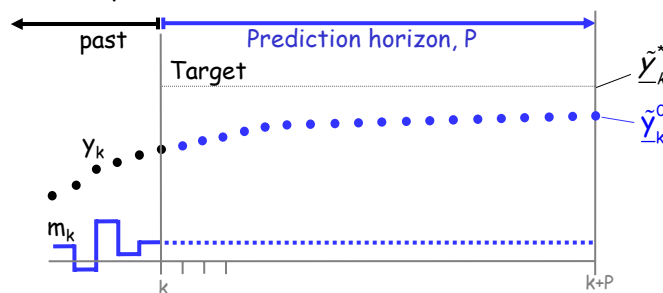
- 1 The process output, y_k , is measured, and used to estimate the process model error using Eq. (11.10).



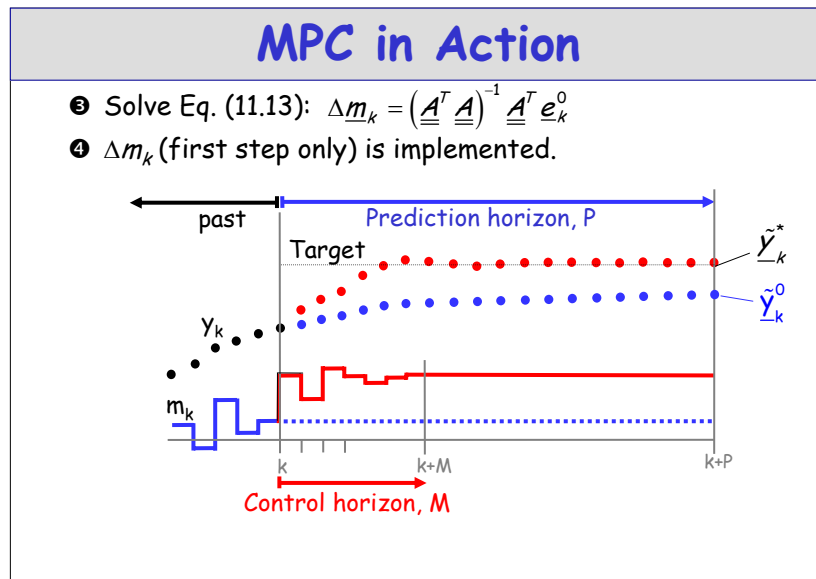
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MPC in Action

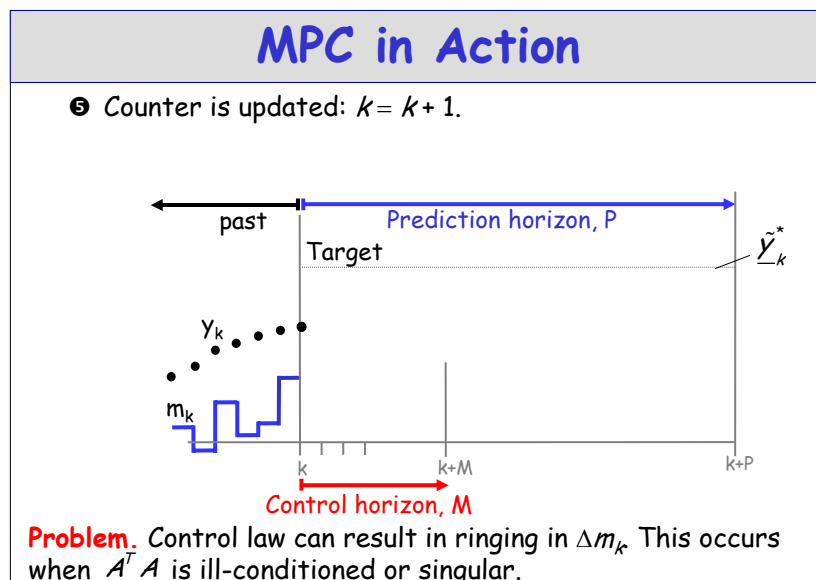
- 2 The open-loop prediction error, $e_k^0 = \tilde{y}_k^* - \tilde{y}_k^0 - \hat{w}_k$, is updated (accounting for changes in setpoint and effect of previous controller moves)



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Dynamic Matrix Control

Solution. Modify the objective function, to penalize for excessive control moves:

$$\phi_{MS} = \left[\underline{e}_k^0 - \underline{A} \cdot \Delta \underline{m}_k \right]^T \left[\underline{e}_k^0 - \underline{A} \cdot \Delta \underline{m}_k \right] + \Delta \underline{m}_k^T \underline{W}_2 \Delta \underline{m}_k \quad (11.14)$$

The inclusion of the positive definite matrix, \underline{W}_2 , in the objective function is referred to as "move suppression."

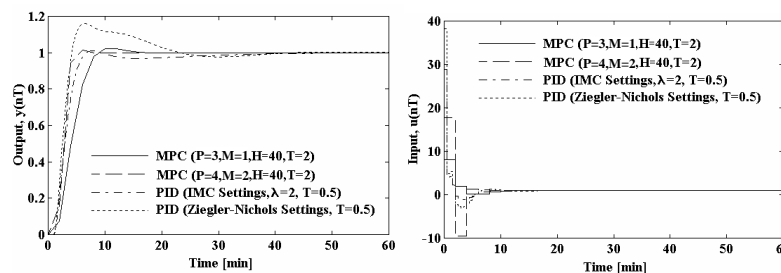
Minimizing with respect to $\Delta \underline{m}_k$ gives: $\Delta \underline{m}_k = \left(\underline{A}^T \underline{A} + \underline{W}_2 \right)^{-1} \underline{A}^T \underline{e}_k^0$

Example. Design MPC/DMC for the process: $p(s) = \frac{1}{(10s+1)(5s+1)} e^{-s}$
Use $T = 2$ min, and $H = 40$, and test: (a) $M = 1, P = 3$, and (b) $M = 2, P = 4$. Compare results with PID tuned by (c) ZN rule, and (d) IMC-PID method. Test the performance of the designs for unit step change in setpoint for nominal process model and for the case with 50% gain and delay uncertainty.

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Comparing MPC with PID (SISO)

Nominal servo responses.

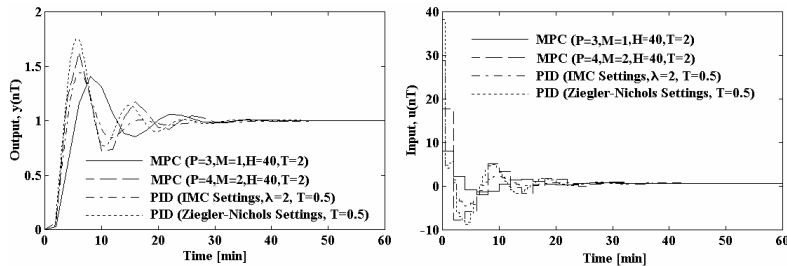


ZN tuning is: $K_c = 4.2, \tau_I = 6, \tau_D = 4$. IMC-PID tuning (with $\lambda = 2$) is: $K_c = 3.75, \tau_I = 15, \tau_D = 3.33$. Note that IMC-PID gives similar performance to MPC, superior to ZN (Is this surprising?)

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Comparing MPC with PID (SISO)

Servo responses with 50% uncertainty in gain and delay.



Increasing M and P leads to more aggressive control action and less robustness.

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MIMO MPC

The results obtained so far are easily extended to cover MIMO systems. Consider for example the 2x2 system described by the following discrete convolution model:

$$\tilde{y}_{1,k+1} = \sum_{i=1}^H a_{11,i} \Delta m_{1,k+1-i} + \sum_{i=1}^H a_{12,i} \Delta m_{2,k+1-i}$$

$$\tilde{y}_{2,k+1} = \sum_{i=1}^H a_{21,i} \Delta m_{1,k+1-i} + \sum_{i=1}^H a_{22,i} \Delta m_{2,k+1-i}$$

This model can be transformed into the standard dynamic form:

$$\underline{E}_k^* = -\underline{A} \cdot \underline{\Delta M}_k + \underline{E}_k^0$$

where \underline{E}_k^* and \underline{E}_k^0 are vectors of length $2P$ and $\underline{\Delta M}_k$ is a vector of length $2M$:

$$\underline{E}_k^* = [e_{1,k+1}^*, e_{1,k+2}^*, \dots, e_{1,k+P}^*, e_{2,k+1}^*, e_{2,k+2}^*, \dots, e_{2,k+P}^*]^T$$

$$\underline{\Delta M}_k = [\Delta m_{1,k}, \Delta m_{1,k+1}, \dots, \Delta m_{1,k+M-1}, \Delta m_{2,k}, \Delta m_{2,k+1}, \dots, \Delta m_{2,k+M-1}]^T$$

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MIMO MPC

In this 2x2 case, the matrix \underline{A} has the following structure:

$$\underline{A} = \begin{bmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{bmatrix} \text{ where each sub-matrix } \underline{A}_{ij} \text{ has the structure:}$$

$$\underline{A}_{ij} = \begin{bmatrix} a_{ij,1} & 0 & 0 & \cdots & 0 \\ a_{ij,2} & a_{ij,1} & 0 & \cdots & 0 \\ a_{ij,3} & a_{ij,2} & a_{ij,1} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{ij,p} & a_{ij,p-1} & a_{ij,p-2} & \cdots & a_{ij,p-M+1} \end{bmatrix}$$

Even more general objective function is:

$$\phi = \underline{E}_k^* \underline{W}_1 \underline{E}_k^* + \Delta \underline{M}_k^T \underline{W}_2 \Delta \underline{M}_k$$

$$\text{Minimizing gives: } \Delta \underline{M}_k = \left(\underline{A}^T \underline{W}_1 \underline{A} + \underline{W}_2 \right)^{-1} \underline{A}^T \underline{W}_1 \underline{E}_k^0$$

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Comparing MPC with PID (MIMO)

Example. Consider the Wood and Berry distillation column:

$$\begin{bmatrix} x_D \\ x_B \end{bmatrix} = \begin{bmatrix} \frac{12.8}{16.7s+1} e^{-s} & \frac{-18.9}{21s+1} e^{-3s} \\ \frac{6.6}{10.9s+1} e^{-7s} & \frac{-19.4}{14.4s+1} e^{-3s} \end{bmatrix} \begin{bmatrix} L \\ V \end{bmatrix}$$

We will compare two MIMO control systems for the process:

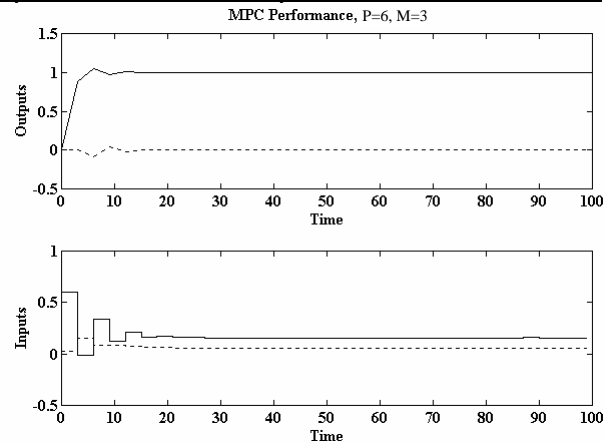
- (a) an MPC design for $H=30$, $P=6$, $M=3$, $T=3$, and $\underline{W}_2 = 0$;
- (b) a diagonal IMC-PID control system, tuned with $\underline{\lambda} = [6, 2.5]$.

Results. The following slides indicate that: (a) Nominally, MPC provides superior performance to decentralized PID, since it decouples the process interactions; (b) It is more sensitive to model uncertainties (50% in gain and delay) than PID.

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Comparing MPC with PID (MIMO)

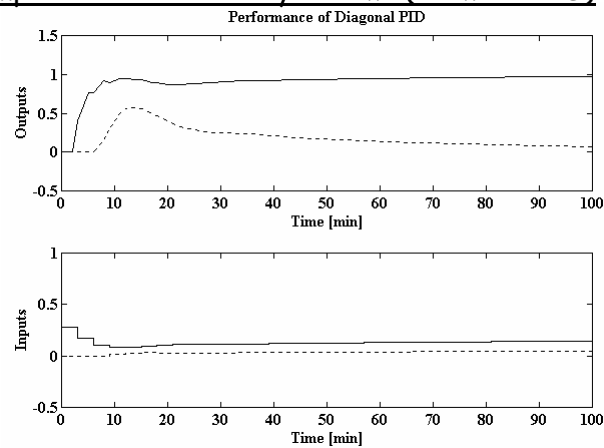
Example: Wood and Berry Column (Nominal MPC)



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Comparing MPC with PID (MIMO)

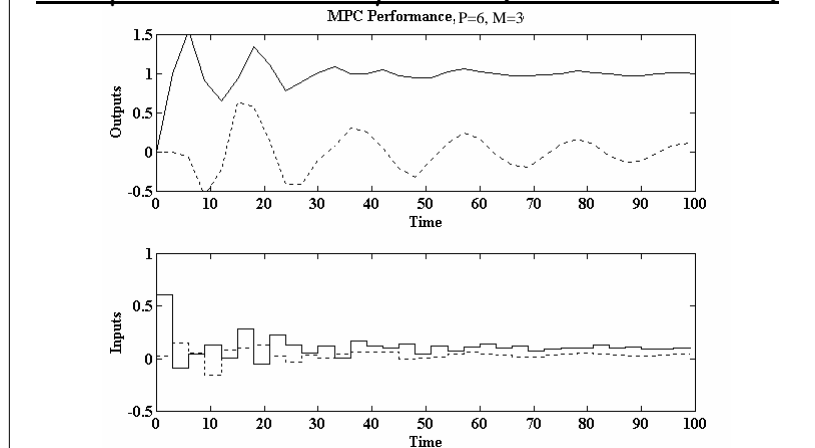
Example: Wood and Berry Column (Nominal PID)



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Comparing MPC with PID (MIMO)

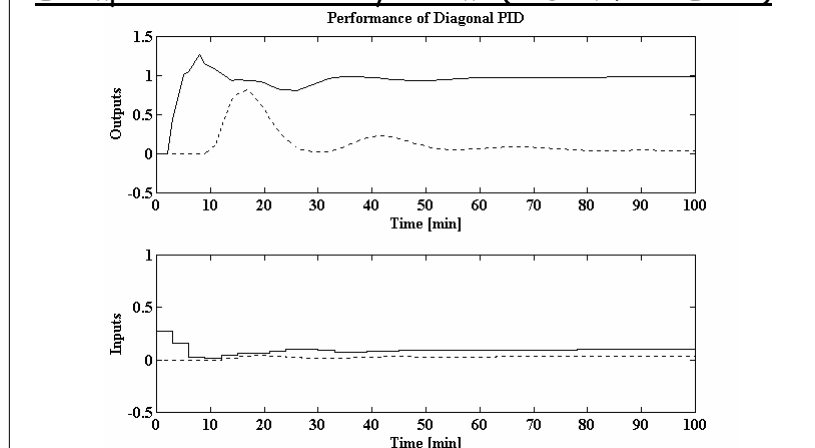
Example: Wood and Berry Column (MPC - Model Error)



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Comparing MPC with PID (MIMO)

Example: Wood and Berry Column (PID - Model Error)



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Tuning the MPC Controller

The following recommendations are often considered when selecting the tunable parameters in MPC:

- ❶ H - model horizon.
 - HT should be selected so that it is greater than the open-loop settling time (defined as 99 or 95% of response). Commonly $20 \leq H \leq 70$.
- ❷ T - sampling period.
 - Should be selected to capture important dynamic information.
- ❸ P and M - prediction and control horizons.
 - Generally, $P > M$. Increasing M increases controller aggressiveness and decreases robustness. A suitable first guess $P \approx t_{60}/T$ (number of samples to reach 60% of open-loop response).
- ❹ W_1 and W_2 - weighting matrices.
 - W_2 , generally defined as a diagonal matrix, permits selectively penalizing particular manipulated variables for MIMO systems ("move suppression"). W_1 allows output scaling to be handled, and is referred to as "equal concern error" in DMC.

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Process Constraints

In practice, the manipulated variables are constrained to physical, or allowable limits. Furthermore, the output variables may also be subject to constraints. Thus, in practice, a constrained optimization problem is solved:

$$\min_{\Delta \underline{M}_k} \phi = \underline{E}_k^{*T} \underline{W}_1 \underline{E}_k^* + \Delta \underline{M}_k^T \underline{W}_2 \Delta \underline{M}_k \quad (11.15)$$

subject to: $\Delta \underline{m}^L \leq \Delta \underline{m}_k \leq \Delta \underline{m}^H$ (move limits)

$\underline{m}^L \leq \underline{m}_k \leq \underline{m}^H$ (manipulated variable bounds)

$\underline{y}^L \leq \underline{y}_k \leq \underline{y}^H$ (output variable bounds)

This type of problem can be solved using quadratic programming. Nonlinear versions are solved using successive quadratic programming (SQP).

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Why Model Predictive Control ?

Real processes are **multivariable** and often involve significant **interaction**. Here's a good analogy to convince your future boss to implement MPC.

Driving a car down Haifa's, Freud Street

- When driving a car, we make coordinated use of:
 - Steering wheel
 - Brake pedal
 - Acceleration pedal
 - Gear stick
- Consider how well you could maneuver a car at speed down Freud Street using only the steering wheel, with all other controls fixed? Equivalent to SISO control...

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Summary

On completing this section, you should:

- ① Be able to explain to a layman how model predictive control (MPC) works.
 - Recall that MPC uses a model in two ways: (a) to **predict** the effects of past moves; (b) to **optimize** future moves.
- ② Understand why MPC can improve on the performance that can be expected from decentralized controllers.
 - Recalling that MPC: (a) implicitly handles constraints and thus maximizes DOF; (b) effectively decouples process interactions.
- ③ Be able to correctly tune MPC to obtain a good response.
- ④ Be able to implement MPC using MATLAB.

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