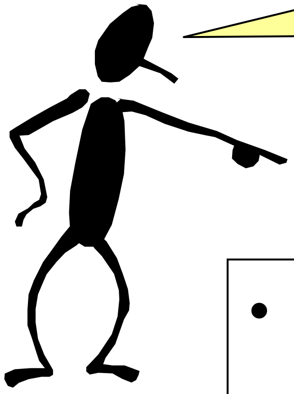
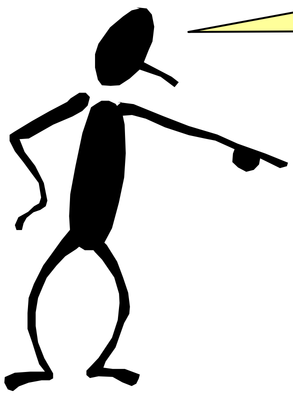
CHAPTER 23: Centralized MPC Control



When I complete this chapter, I want to be able to do the following.

- Explain centralized control
- Build a discrete process model
- Explain the trajectory optimization for MPC
- Explain the advantages of MPC
- Describe criteria for selecting MPC over multiloop control

CHAPTER 23: Centralized MPC Control

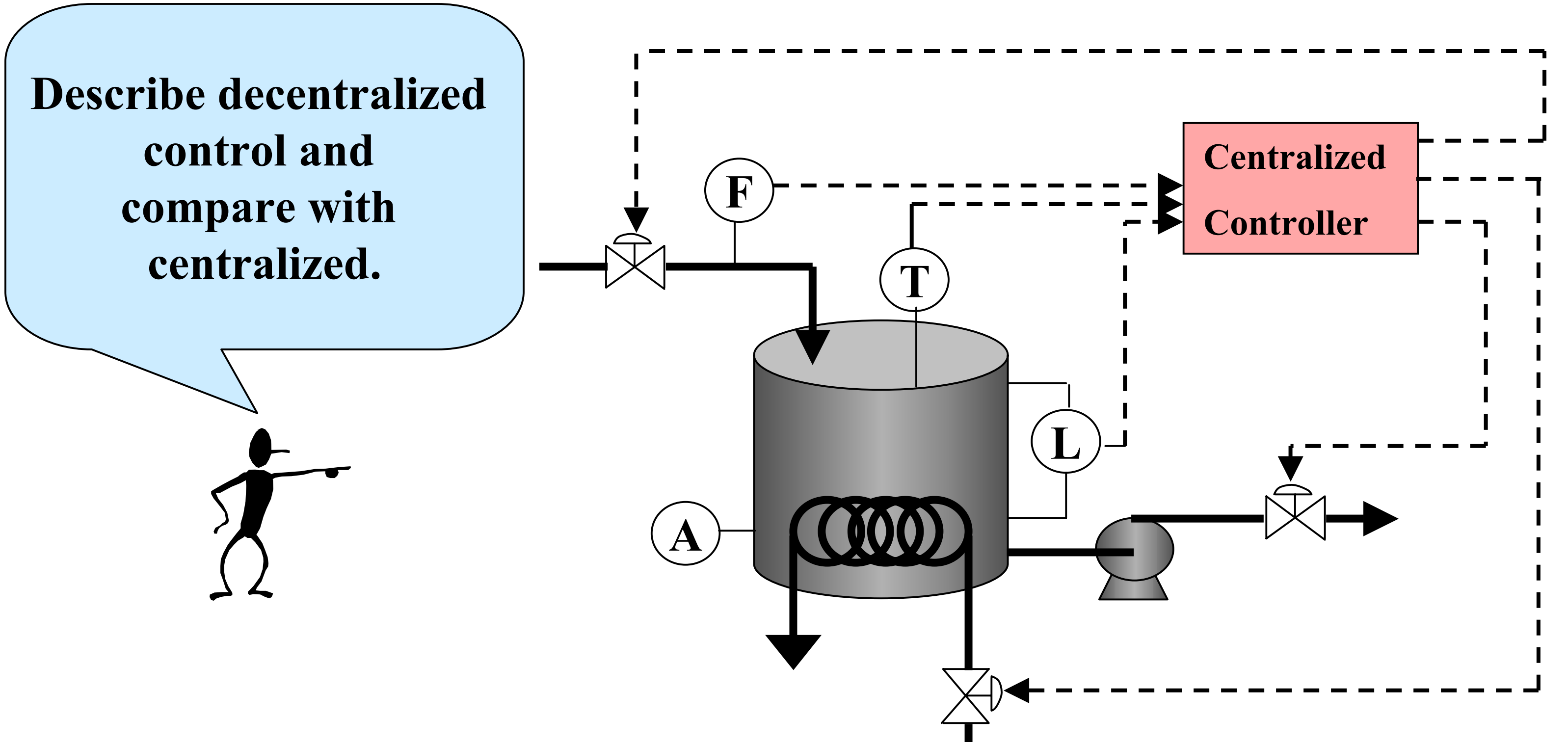


Outline of the lesson.

- Review IMC structure and principles
- Develop MPC for SISO
- Generalize MPC for MIMO
- Discuss important enhancements
- Explain key insights
- Develop application guidelines

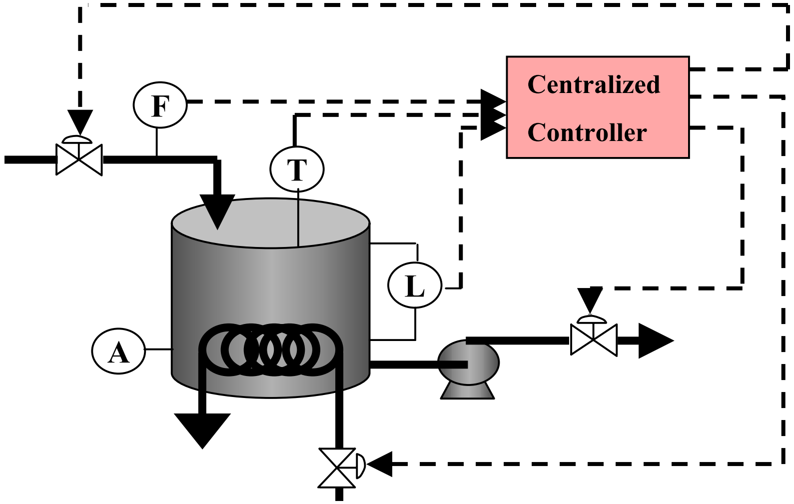
CHAPTER 23: Centralized MPC Control

Centralized Control uses all CV measurements to calculate all manipulated variables simultaneously.



CHAPTER 23: Centralized MPC Control

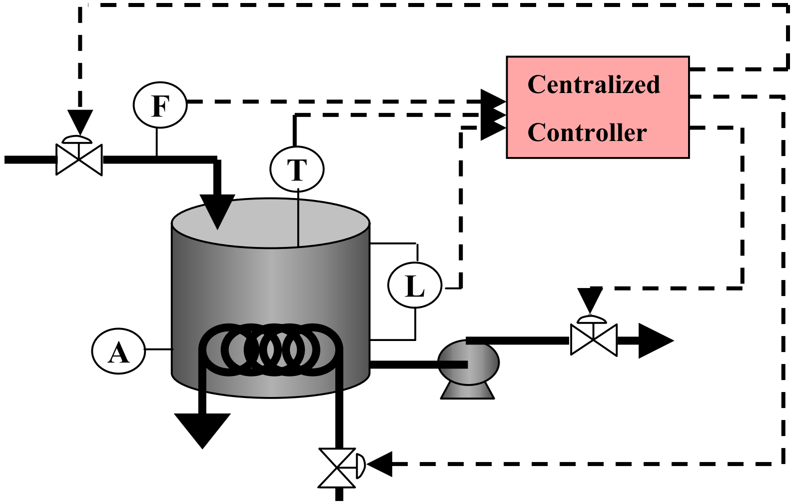
Centralized Control is applied to control systems with interaction.



- All process information is used by the controller
- Better control performance is possible
- Substantial increases in controller complexity and real-time computations

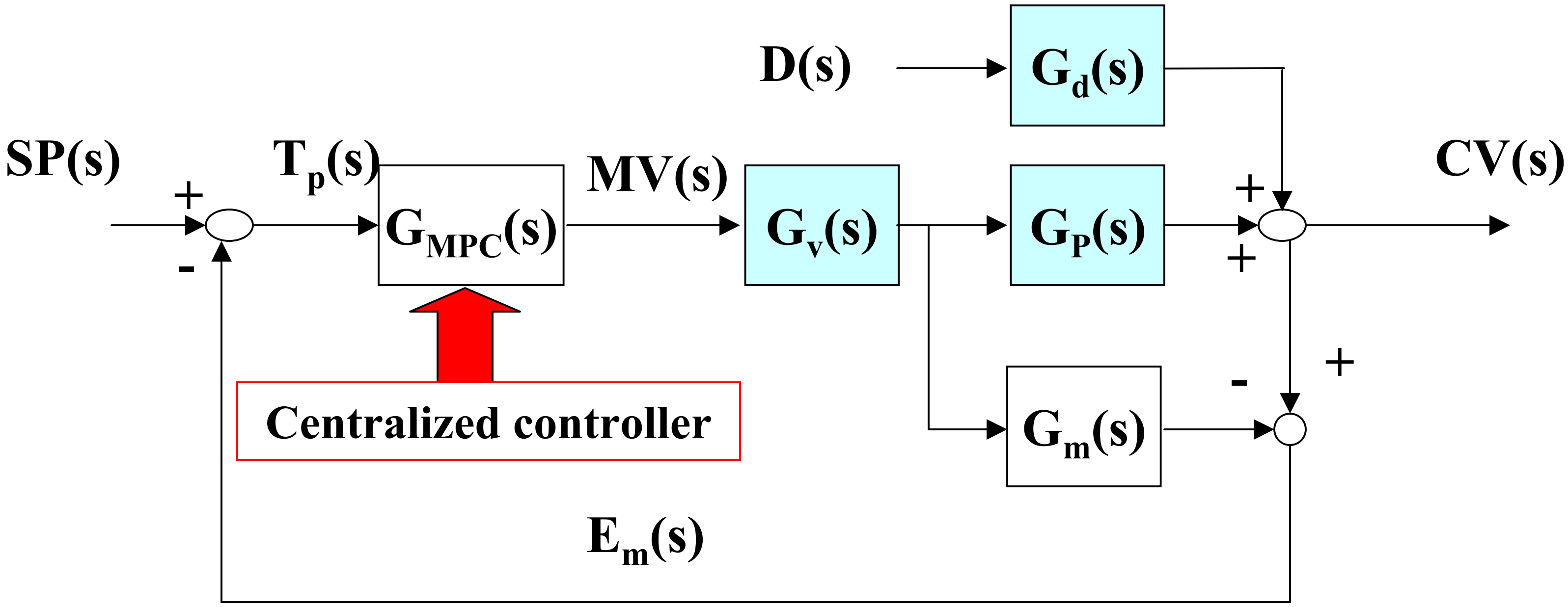
CHAPTER 23: Centralized MPC Control

Centralized Control developed in the 1960's with major enhancements around 1980.



- Original use was limited to few industries and highly skilled practitioners
- Now widely applied in many industries
- Good software facilitates applications
- We will discuss the Dynamic Matrix Control approach, using the IMC control structure

CHAPTER 23: Centralized MPC Control



Let's recall the IMC structure

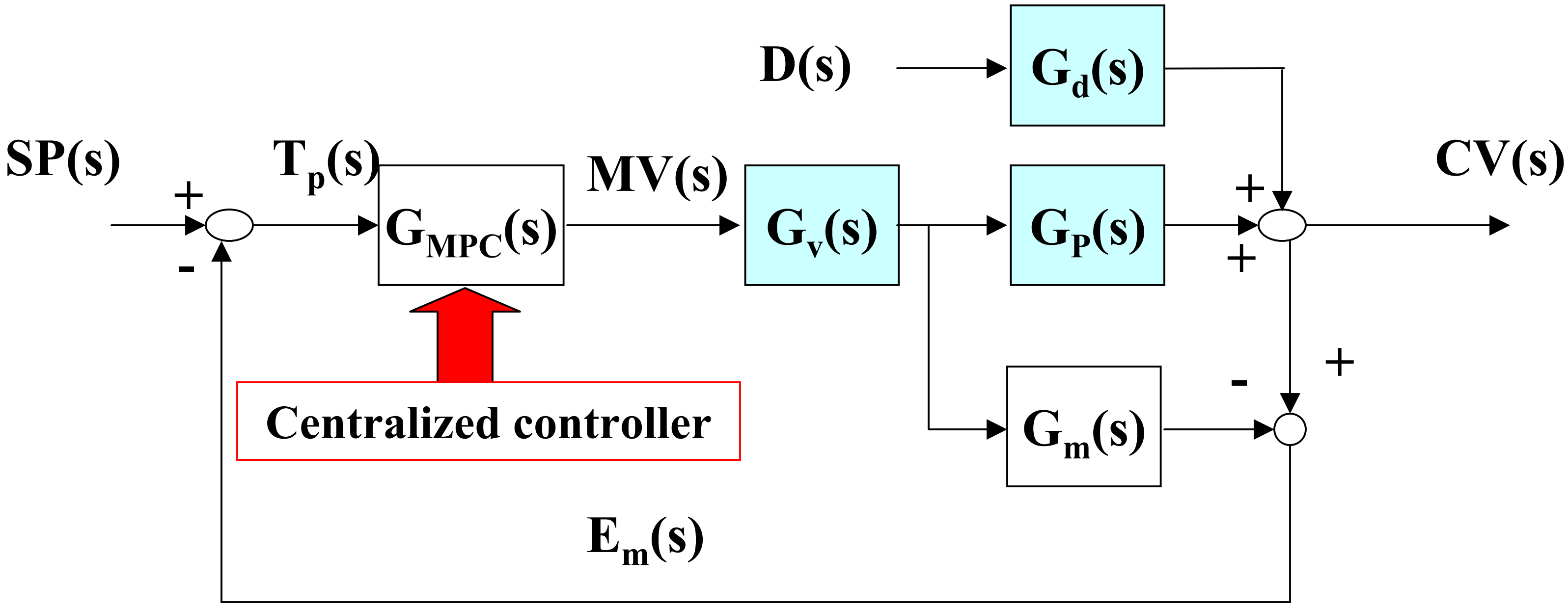
<u>Transfer functions</u>

$G_{MPC}(s)$ = controller
$G_v(s)$ = valve
$G_P(s)$ = feedback process
$G_m(s)$ = model
$G_d(s)$ = disturbance process

<u>Variables</u>

CV(s) = controlled variable
$CV_m(s)$ = measured value of CV(s)
D(s) = disturbance
$E_m(s)$ = model error
MV(s) = manipulated variable
SP(s) = set point
$T_p(s)$ = set point corrected for model error

CHAPTER 23: Centralized MPC Control



The variables are vectors

$$CV(s) = \begin{bmatrix} CV_1(s) \\ CV_2(s) \\ \ldots \\ \ldots \\ CV_N(s) \end{bmatrix}$$

The transfer functions are matrices

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & .. & .. & G_{1M}(s) \\ G_{21}(s) & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. \\ G_{N1}(s) & .. & .. & .. & G_{NM}(s) \end{bmatrix}$$

CHAPTER 23: Centralized MPC Control

From Chapter 19, we know that the IMC structure must obey the following criteria.

1. **The controller steady-state gain must be the inverse of the model gain.**

$$G_{MPC}(0) = [G_m(0)]^{-1} \qquad \text{with } s = 0 \Rightarrow \text{ steady state}$$

2. **Perfect control requires an exact inverse.**

$$G_{MPC}(s) = [G_m(s)]^{-1}$$

3. **Realistic control cannot "look into the future" or invert models that lead to unstable controllers.**

CHAPTER 23: Centralized MPC Control

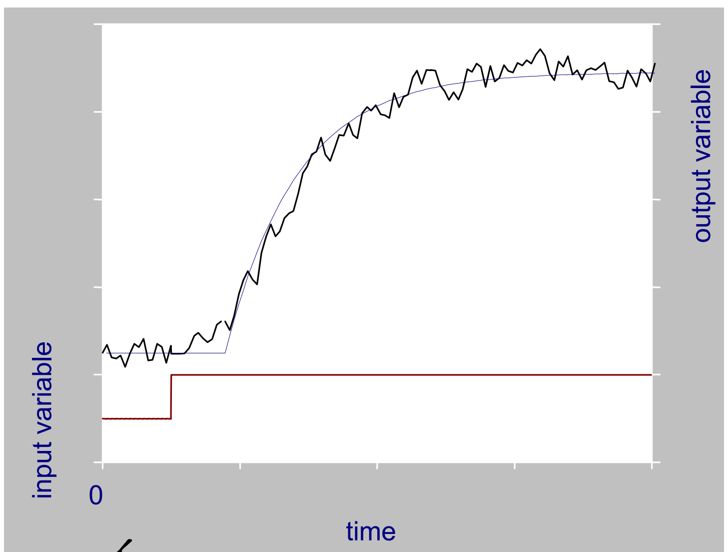
What does the MPC Controller calculate?

- **It determines the best (optimal) trajectory from the current conditions to the set point.**
- **To perform this calculation using the DMC approach, we need a model that gives predictions of discrete values, rather than a continuous transfer function.**
- **Therefore, we start with a modified modelling approach.**

CHAPTER 23: Centralized MPC Control

Modelling approach: Although we could use use fundamental models, the basis for nearly all applications are empirical models.

Empirical identification

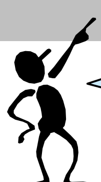


Continuous Model using process reaction curve or statistics

$$G(s) = \frac{1.0e^{-5s}}{5s+1}$$

Sampled output model for unit step with Δt = 2.5 minutes

Sample number	
0	0 (Step here)
1	0
2	0
3	0.394
4	0.632
5	0.777
6	0.864
7	0.918
8	0.950
9	0.970
10	0.982
11	0.989

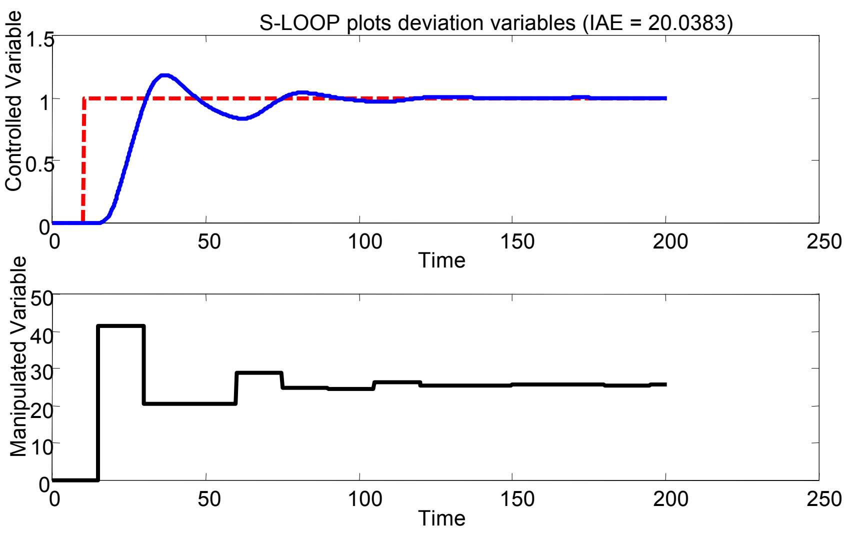


Could we use the data from the experiment as the model?

CHAPTER 23: Centralized MPC Control

Modelling approach:

1. **Inputs can be represented as a series of steps.**
2. **The effect of a step of magnitude ΔMV is the magnitude times the unit step model.**
3. **The output effect of a sequence of inputs is the sum of each input effect.**



CHAPTER 23: Centralized MPC Control

Modelling approach: Directly performed using matrices.

Step model, a_k

0	0
1	0
2	0
3	0.394
4	0.632
5	0.777
6	0.864
7	0.918
8	0.950
9	0.970
10	0.982
11	0.989

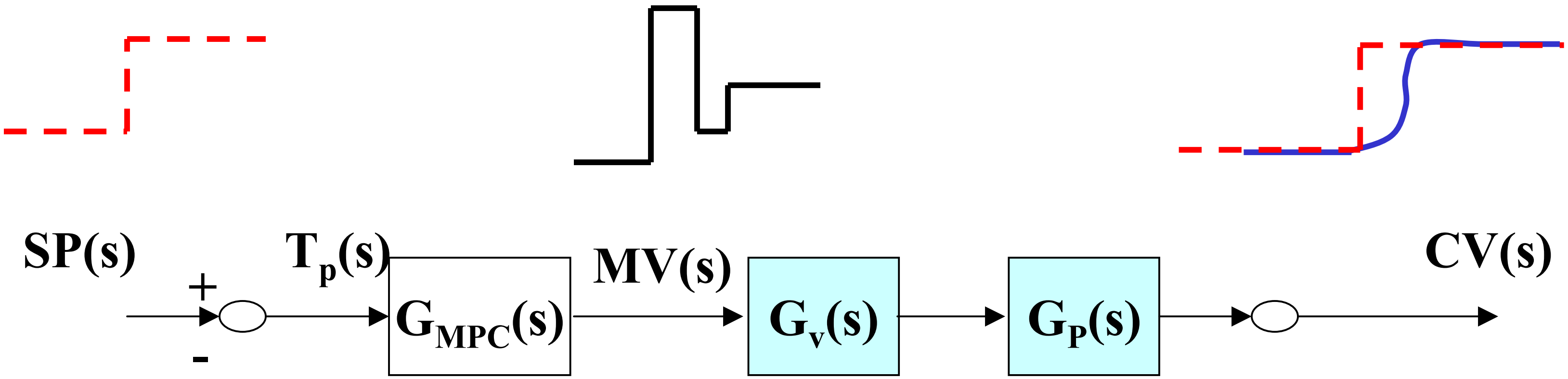
$$\begin{bmatrix} a_1 & 0 & 0 & 0 \\ a_2 & a_1 & 0 & 0 \\ a_3 & a_2 & a_1 & 0 \\ \ldots & .. & .. & .. \end{bmatrix} \begin{bmatrix} \Delta MV_1 \\ \Delta MV_2 \\ \Delta MV_3 \\ .. \end{bmatrix} = \begin{bmatrix} CV_1 \\ CV_2 \\ CV_3 \\ .. \end{bmatrix}$$

Dynamic Matrix → $\mathbf{A} \quad \Delta \mathbf{MV} = \mathbf{CV}$

Notes:
1. **The MV& CV subscripts represent samples (different times)**
2. **The CV values are deviation from initial steady state**
3. **Some time after the input step, the process achieves steady state and the model values (a_k) are constant at K_p**

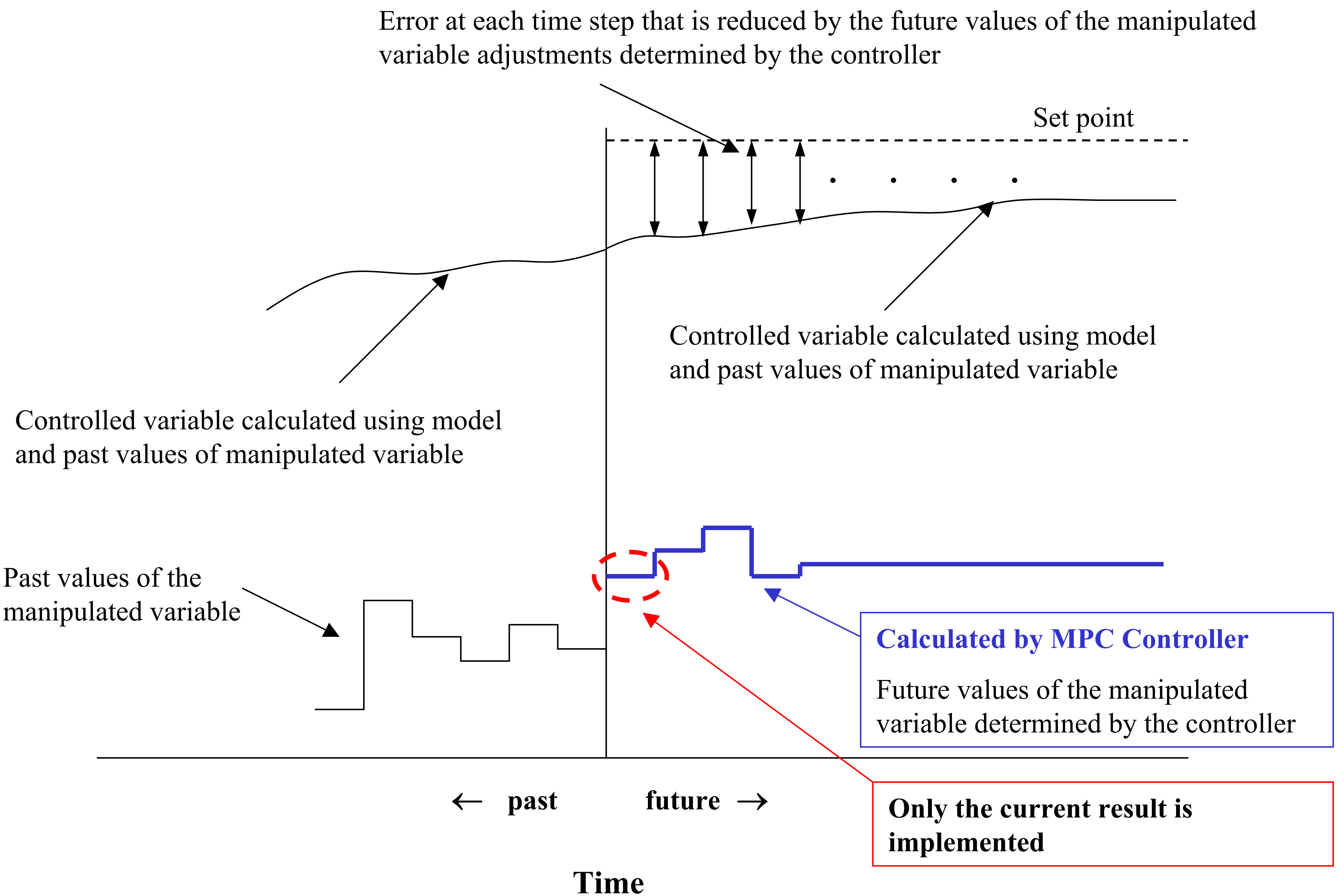
CHAPTER 23: Centralized MPC Control

Basic controller calculation: Open-loop trajectory



The controller calculates the manipulations in the future that will achieve the control objective - which for now is closely tracking the set point.

CHAPTER 23: Centralized MPC Control



CHAPTER 23: Centralized MPC Control

Controller calculation: Controller minimizes sum of squares of error.

First, calculate the response if no future moves occurred, CV^f. This includes the affects of past moves.

$$A(\Delta MV_{past}) = CV^f$$

Second, calculate the error if no future moves occurred, E^f.

$$SP - CV^f = E^f$$

Third, calculate the manipulation to minimize the goal given below.

$$\sum_{samples=i} (E_i^f - CV_i^c)^2$$

with CV^c the effect of the controller manipulation at the current and future times

CHAPTER 23: Centralized MPC Control

Controller calculation: Controller minimizes sum of squares of error.

Means minimize by adjusting values of ΔMV^c

$$\min_{\Delta MV^c} \sum_{samples=i} (E_i^f - CV_i^c)^2$$

$$subject\ \ to$$

$$A\ \ \Delta MV^c = CV^c$$

Model relates ΔMV^c to CV^c

Quadratic objective with linear equations; can be solved

$$K_{DMC} = (A^T A)^{-1} A^T$$

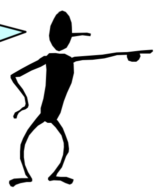
$$\left[\Delta MV^c\right] = \left[K_{DMC}\right]\left[E^f\right]$$

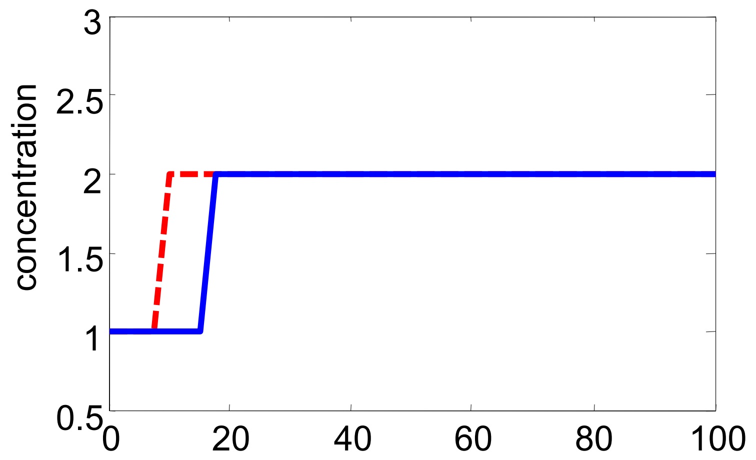
Solution is least squares equations, involving solution to linear equations offline and simple matrix-vector product online.

CHAPTER 23: Centralized MPC Control

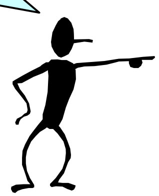
Controller calculation: Example for single I/O controller with perfect model.

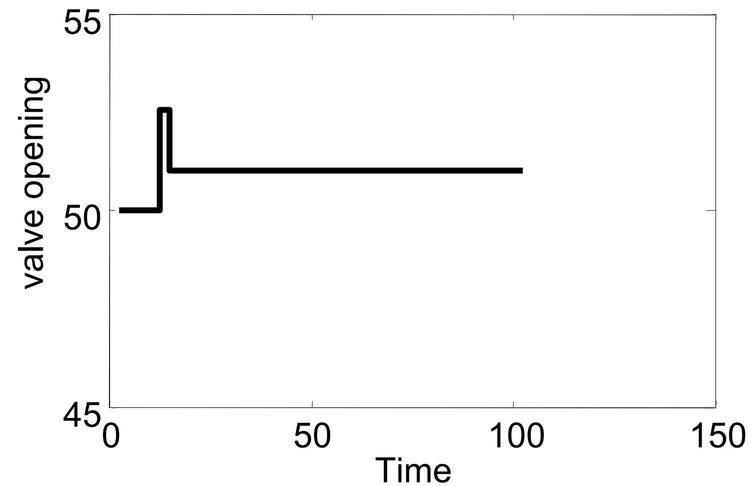
Why didn't the CV track the set point exactly?





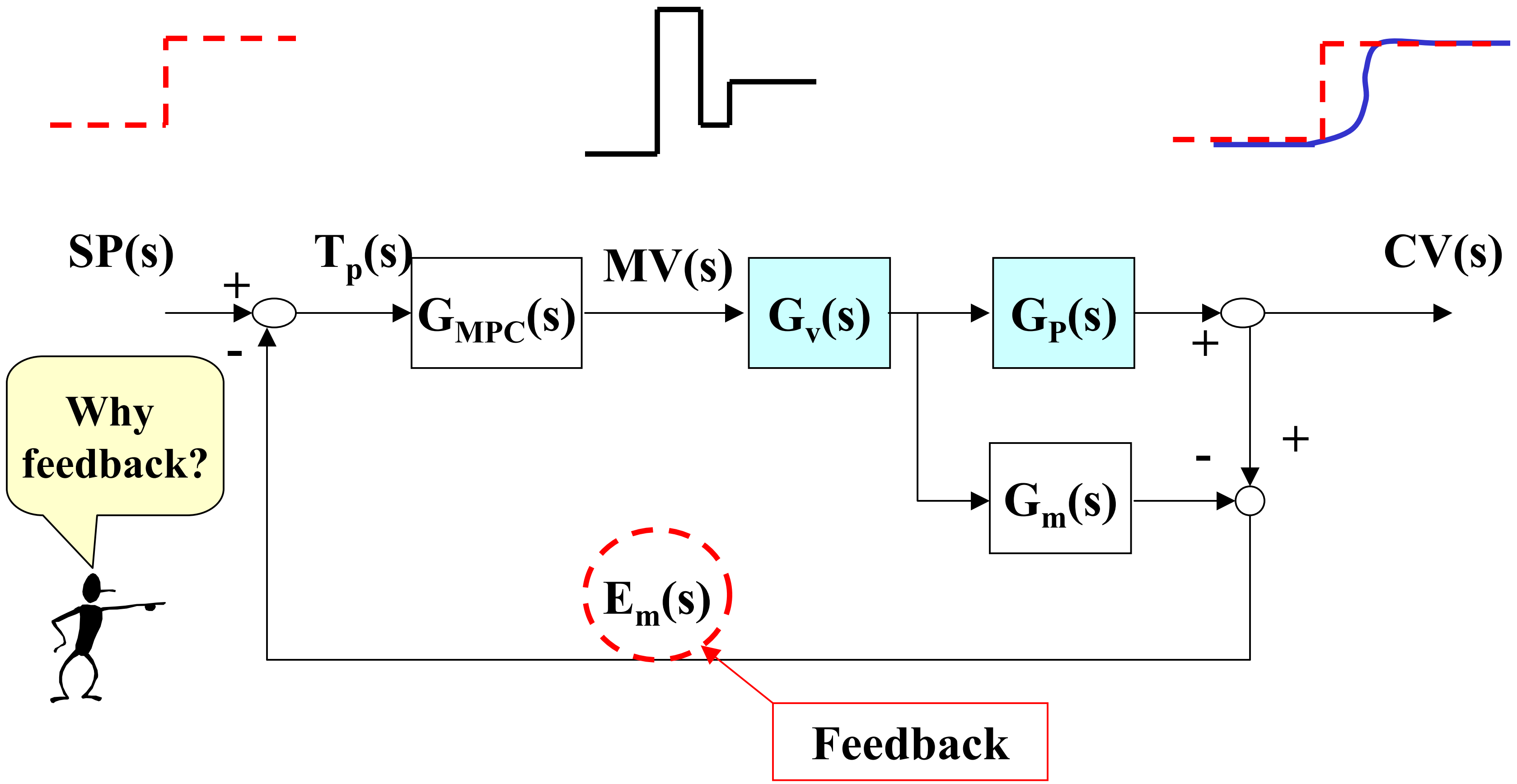
We calculated four steps, but only two changed; why?



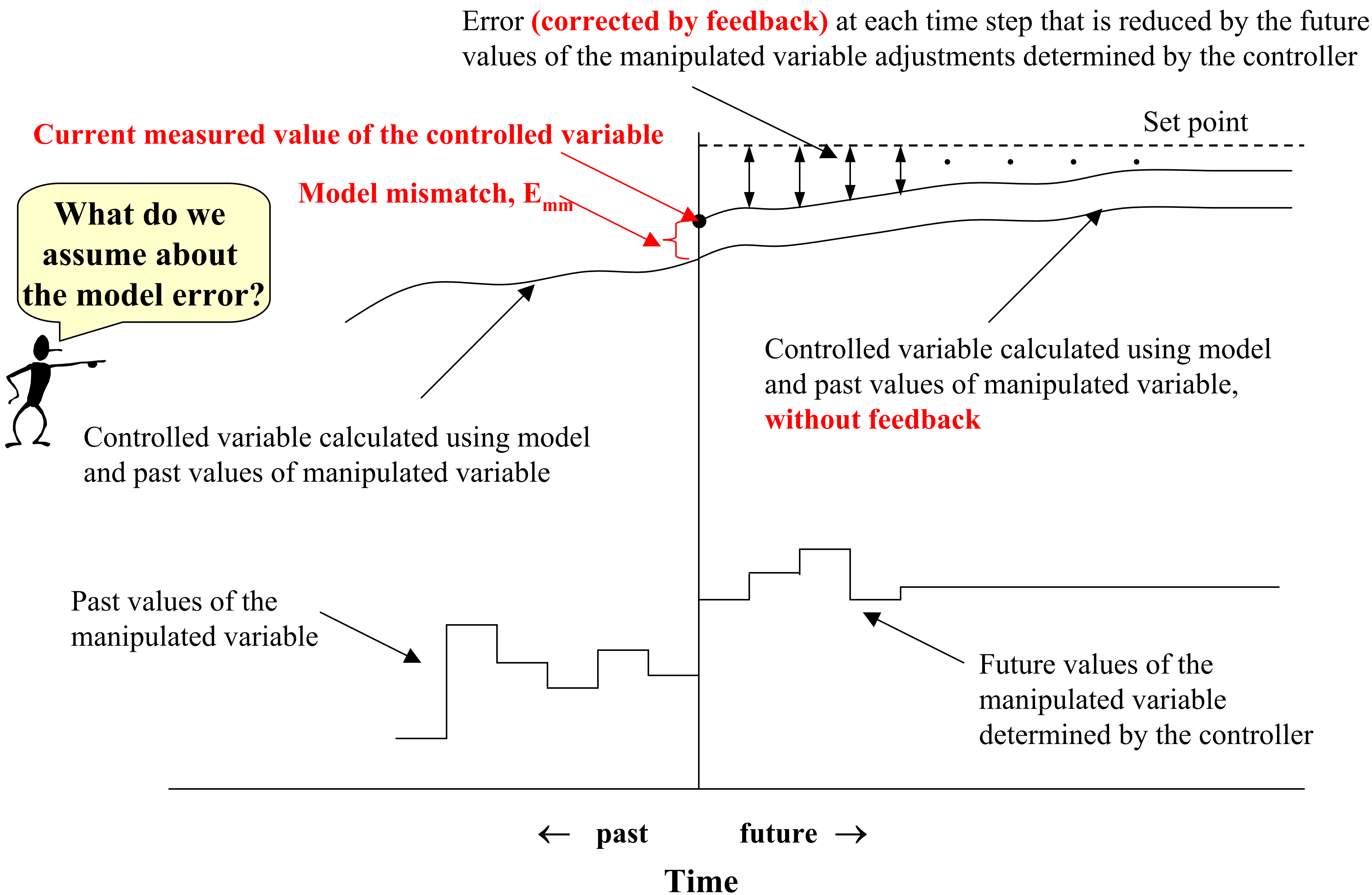


CHAPTER 23: Centralized MPC Control

Controller calculation: We have to added feedback!

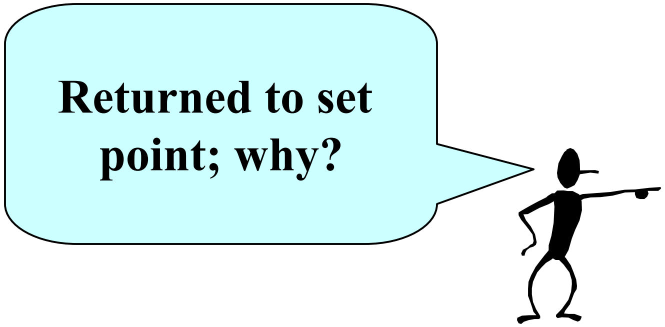


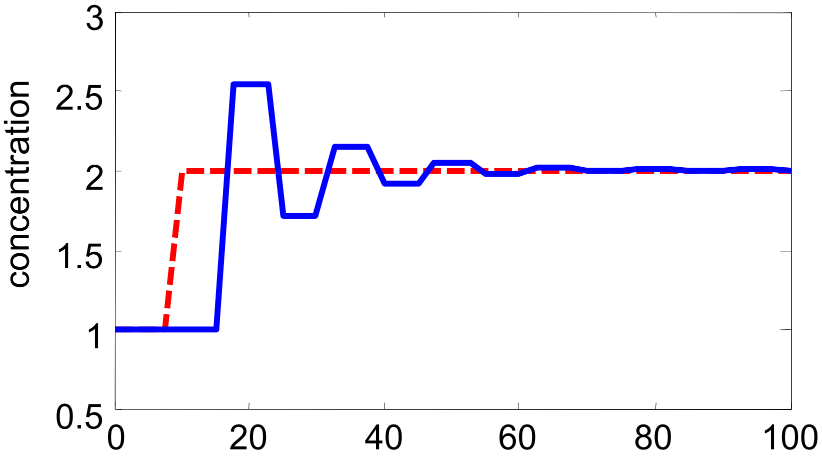
CHAPTER 23: Centralized MPC Control

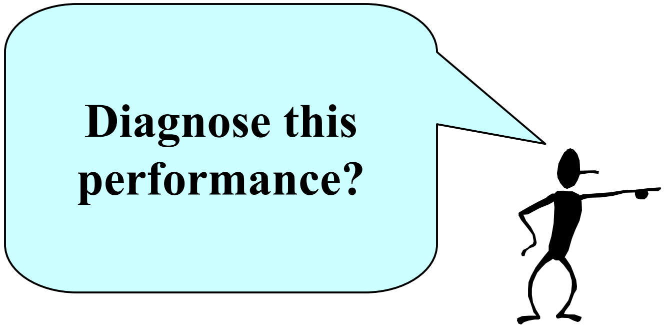


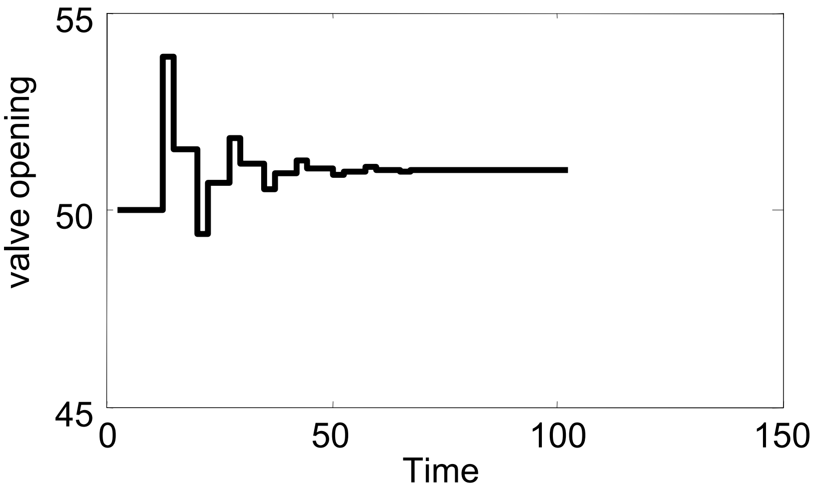
CHAPTER 23: Centralized MPC Control

<u>Controller calculation</u>: Example for single I/O controller with feedback; model error of 35% in gain only.



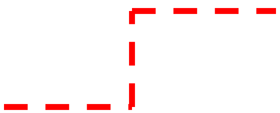


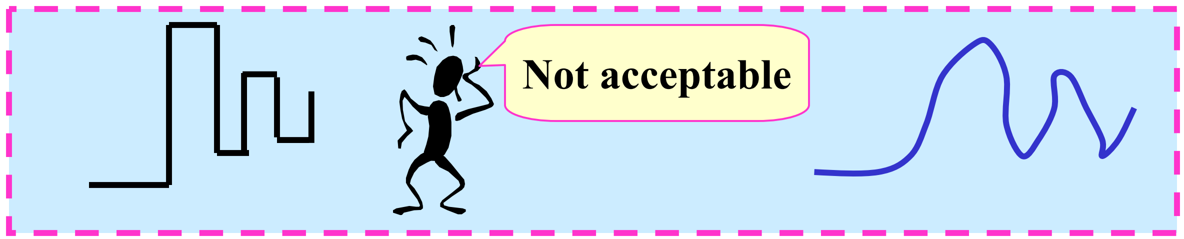


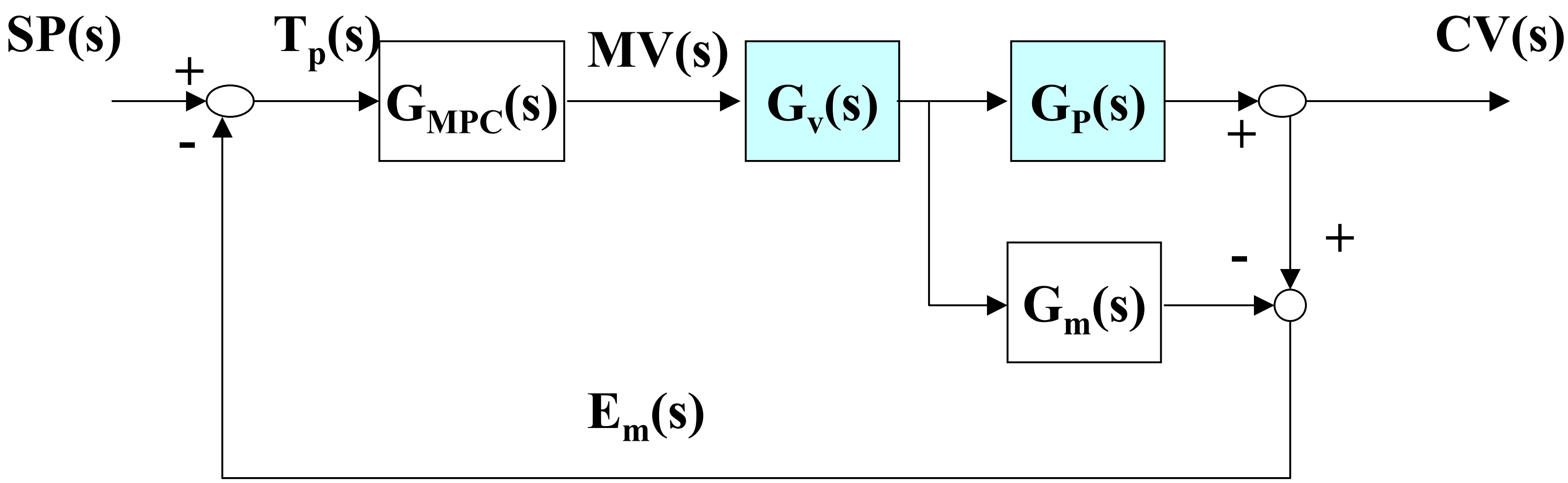


CHAPTER 23: Centralized MPC Control

Controller calculation: We have be able to tune the controller! We want robustness and moderate MV adjustments.







CHAPTER 23: Centralized MPC Control

Controller calculation: Controller minimizes sum of squares of error with penalty on MV moves (ΔMV^2).

$$\min_{\Delta MV^c} \sum_{samples=i} ww(E_i^f - CV_i^c)^2 + \sum_{samples} qq(\Delta MV_i)^2$$

$$subject\ \ to$$

$$A\ \ \Delta MV^c = CV^c$$

Quadratic objective with linear equations; can be solved

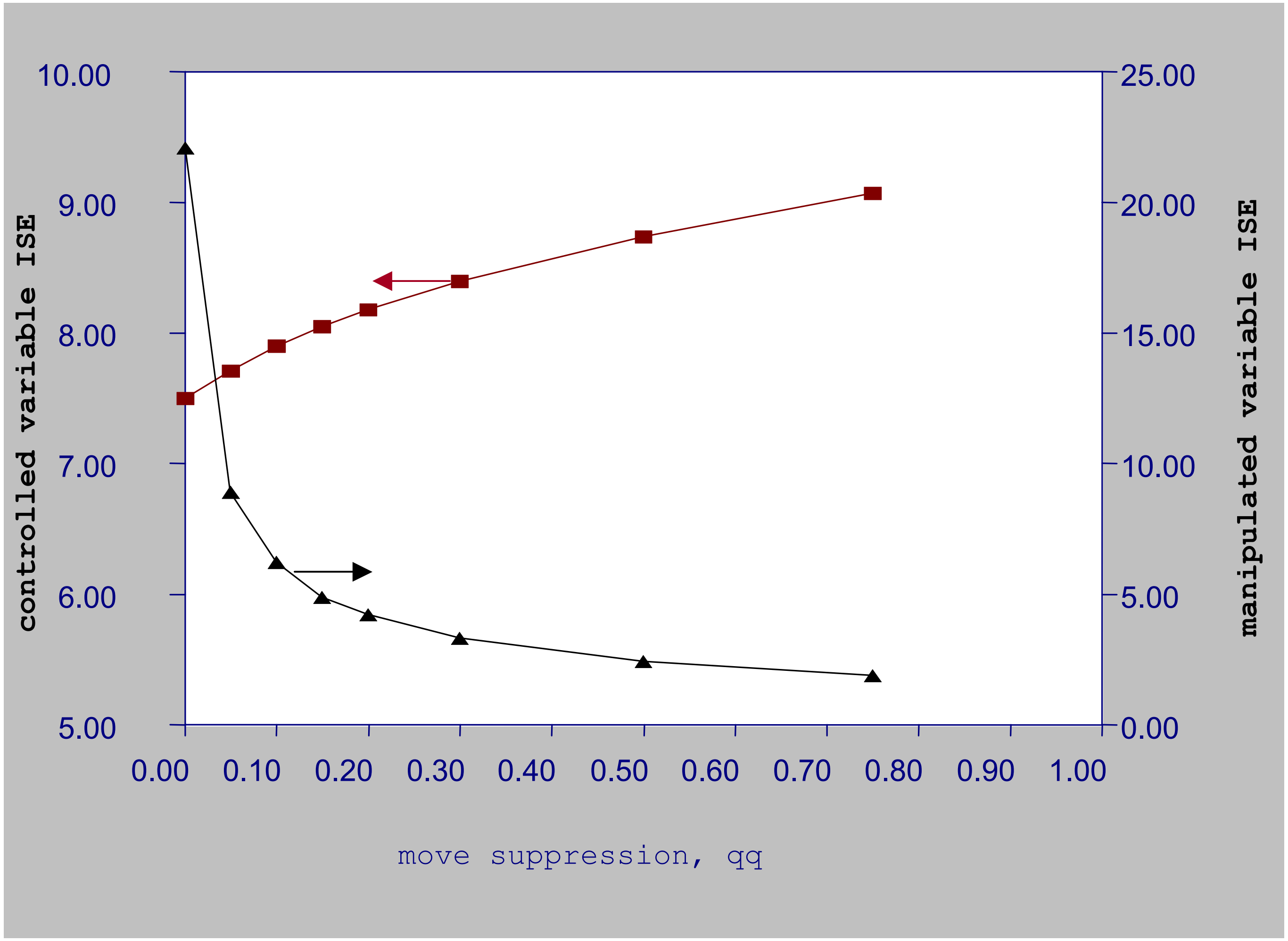
$$K_{DMC} = (A^T [WW] A + QQ)^{-1} A^T [WW]$$

$$\left[\Delta MV^c\right] = \left[K_{DMC}\right]\left[E^f\right]$$

Solution is least squares equations, involving solution to linear equations offline and simple matrix-vector product online.

CHAPTER 23: Centralized MPC Control

Tuning: **Increasing the move penalty (qq/ww) reduces the manipulation magnitude rapidly with small increase in CV variation.**



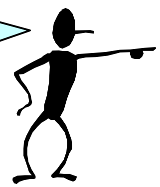
Example results for SISO system in example.

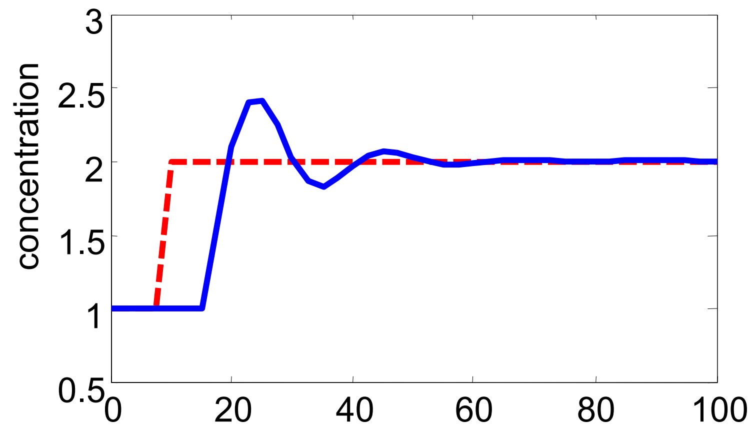
(ww = 1)

CHAPTER 23: Centralized MPC Control

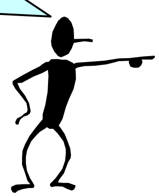
Controller calculation: Example for single I/O controller with feedback; model error of 35% in gain only and with ww = 1.0 and qq = 0.20.

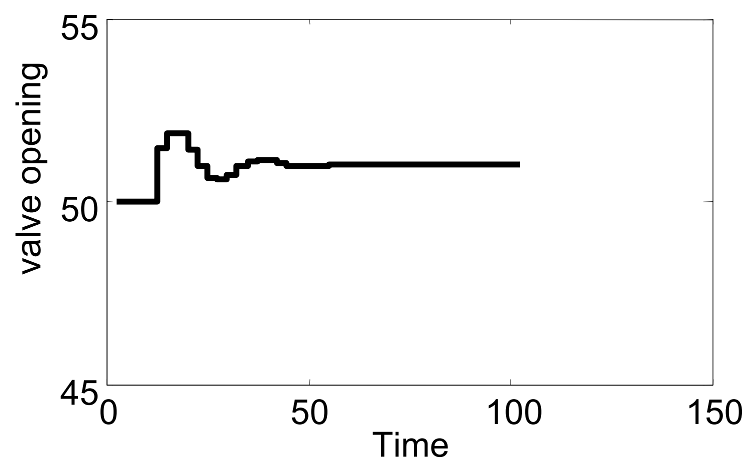
Diagnose and recommend fine tuning.





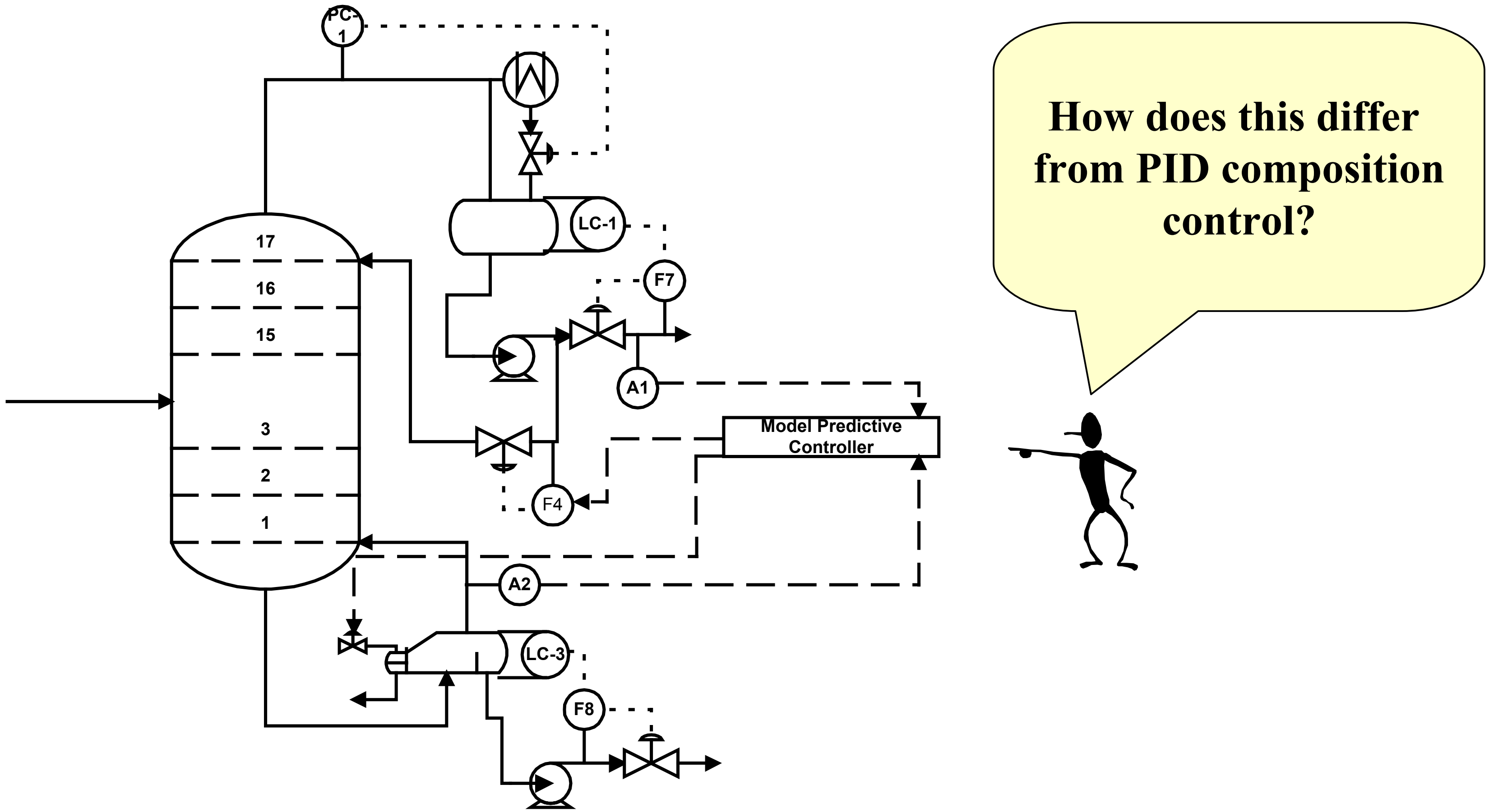
Diagnose and recommend fine tuning.





CHAPTER 23: Centralized MPC Control

A great advantage of MPC is the direct application to centralized multivariable control using the same principles.



CHAPTER 23: Centralized MPC Control

Modelling approach: Must include effects from every MV to every CV.

$$
\begin{bmatrix}
A_{11} & A_{12} & \cdot\cdot & \cdot\cdot \\
A_{21} & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
A_{31} & \cdot\cdot & \cdot\cdot & \cdot\cdot \\
\cdots & \cdot\cdot & \cdot\cdot & A_{NM}
\end{bmatrix}
\begin{bmatrix}
[\Delta MV_1] \\
[\Delta MV_2] \\
[\Delta MV_3] \\
\cdot\cdot
\end{bmatrix}
=
\begin{bmatrix}
[CV_1] \\
[CV_2] \\
[CV_3] \\
\cdot\cdot
\end{bmatrix}
$$

Dynamic Matrix → **A** **ΔMV** **=** **CV**

Notes: **1. A_{ij} is the dynamic matrix between output i and input j**

2. $[\Delta MV_j]$ and $[CV_i]$ are vectors of the values for each variable over the horizon

CHAPTER 23: Centralized MPC Control

Controller calculation: Controller minimizes sum of squares of all errors with penalty on all MV moves.

$$\min_{\Delta MV^c} \sum_{\substack{output \\ variables}} \sum_{samples=i} ww(E_i^f - CV_i^c)^2 + \sum_{\substack{manip. \\ variables}} \sum_{samples} qq(\Delta MV_i)^2$$

$$subject\ \ to$$

$$A\ \ \Delta MV^c = CV^c$$

Quadratic objective with linear equations; can be solved

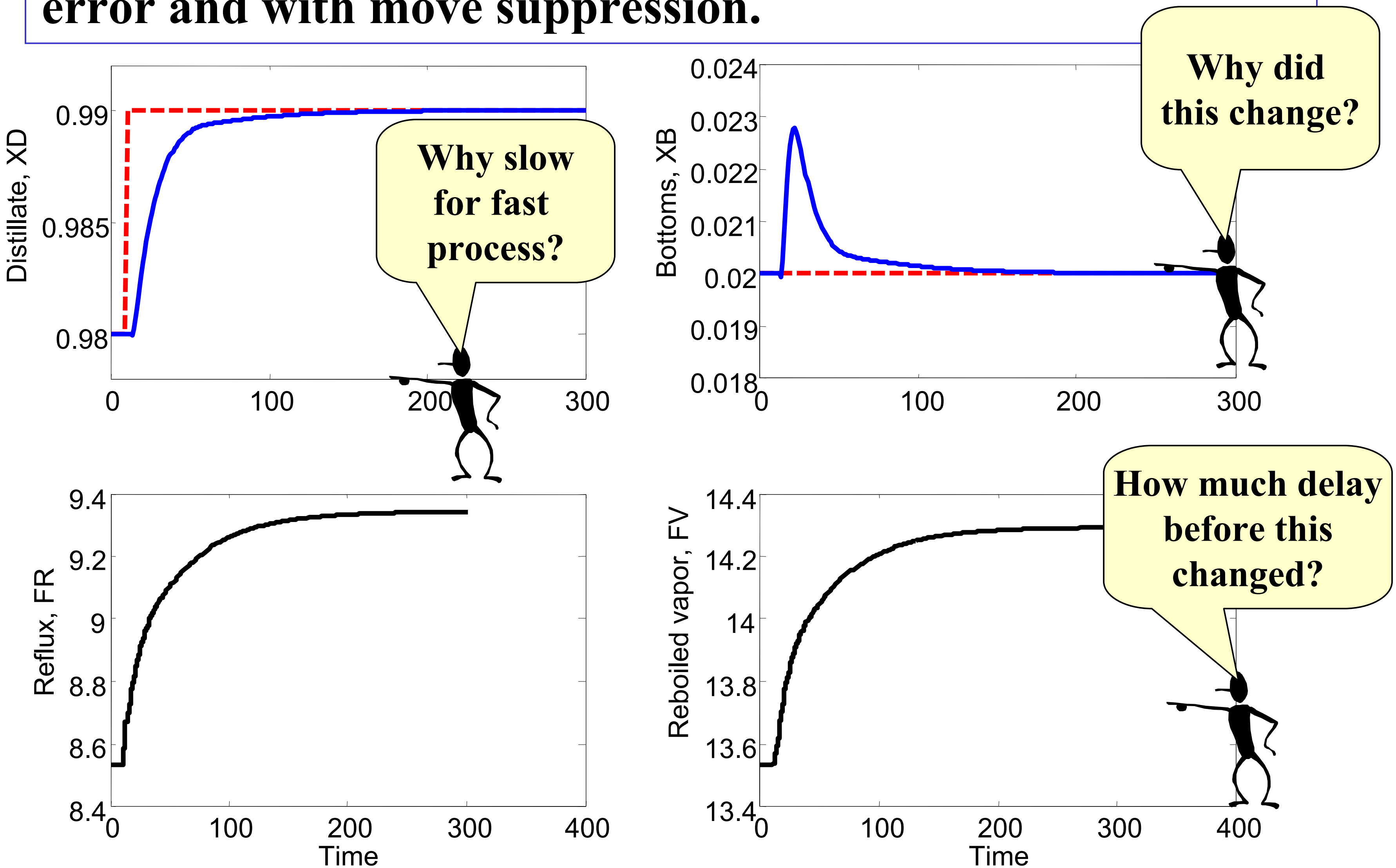
$$K_{DMC} = (A^T[WW]A + QQ)^{-1} A^T [WW]$$

$$\left[\Delta MV^c\right] = \left[K_{DMC}\right]\left[E^f\right]$$

Solution is least squares equations, involving solution to linear equations offline and simple matrix-vector product online.

CHAPTER 23: Centralized MPC Control

Controller calculation: Sample distillation without model error and with move suppression.



CHAPTER 23: Centralized MPC Control

Controller calculation: The following parameters are adjusted by the engineer.

Δt	The controller execution period
NN	The controlled variable horizon, which should be the time to reach steady state
MM	The manipulated variable horizon (≈NN/4)
ww	The weight on each controlled variable; can be used to represent different relative importances
qq	The individual manipulated variable move suppressions; these have units!

CHAPTER 23: Centralized MPC Control

Controller calculation: An important enhancement includes possible limits on CVs and MVs.

$$\min_{\Delta MV^c} \sum_{\substack{output\\ variables}} \sum_{samples=i} ww(E_i^f - CV_i^c)^2 + \sum_{\substack{manip.\\ variables}} \sum_{samples} qq(\Delta MV_j)^2$$

$$\sum \sum vv(D_i^+ + D_i^-)^2$$

Penalty for any violation of controlled variables bounds, D≥0

subject to

$$A\ \Delta MV^c = CV^c$$

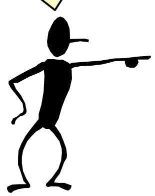
$$MV_{\min} \le \sum \Delta MV_j \le MV_{\max} \quad \text{for every time step}$$

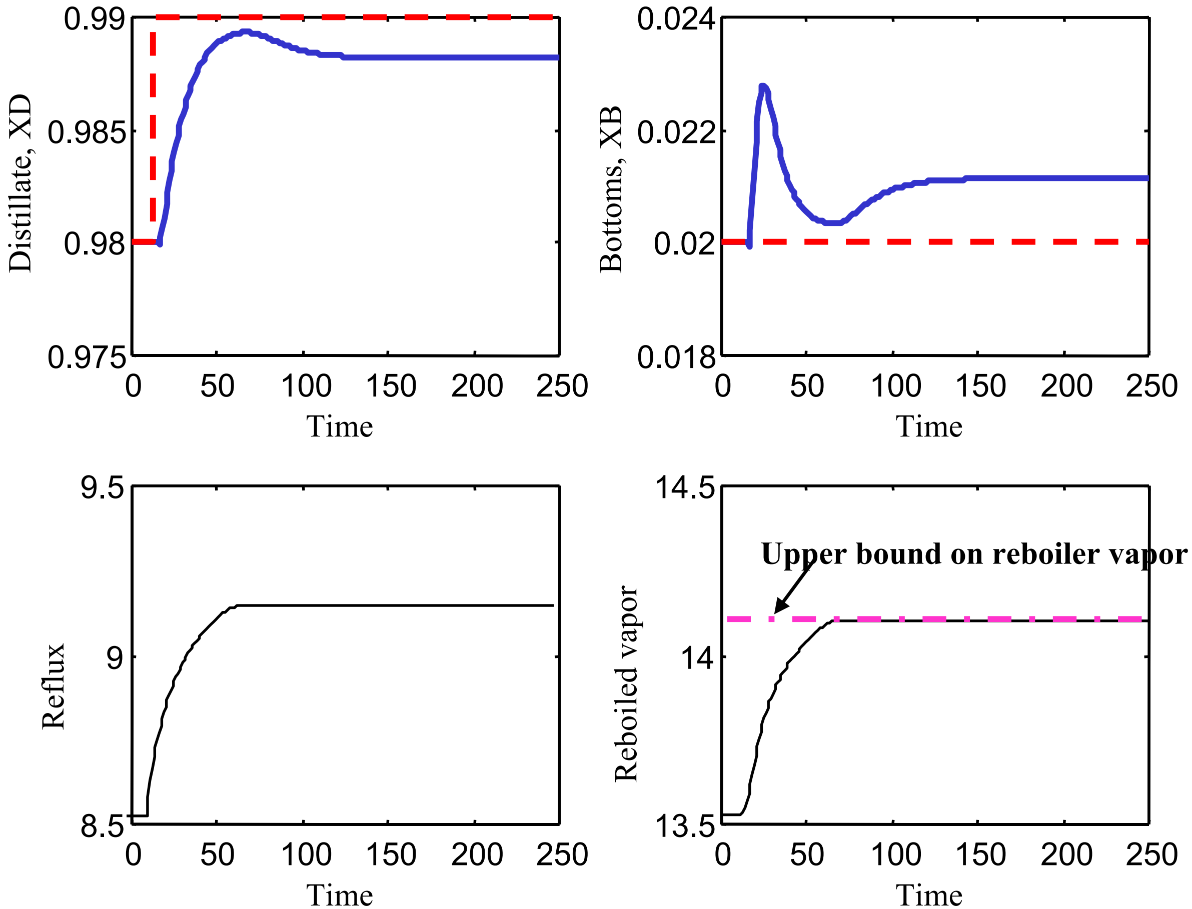
Hard bounds on the manipulated variables

$$CV_{\min} \le \sum (CV_i^c + CV_i^f + D_i^+ - D_i^-) \le CV_{\max} \quad \text{for every time step}$$

Controller calculation: **Sample distillation without model error, move suppression, and bound on manipulated variable.**

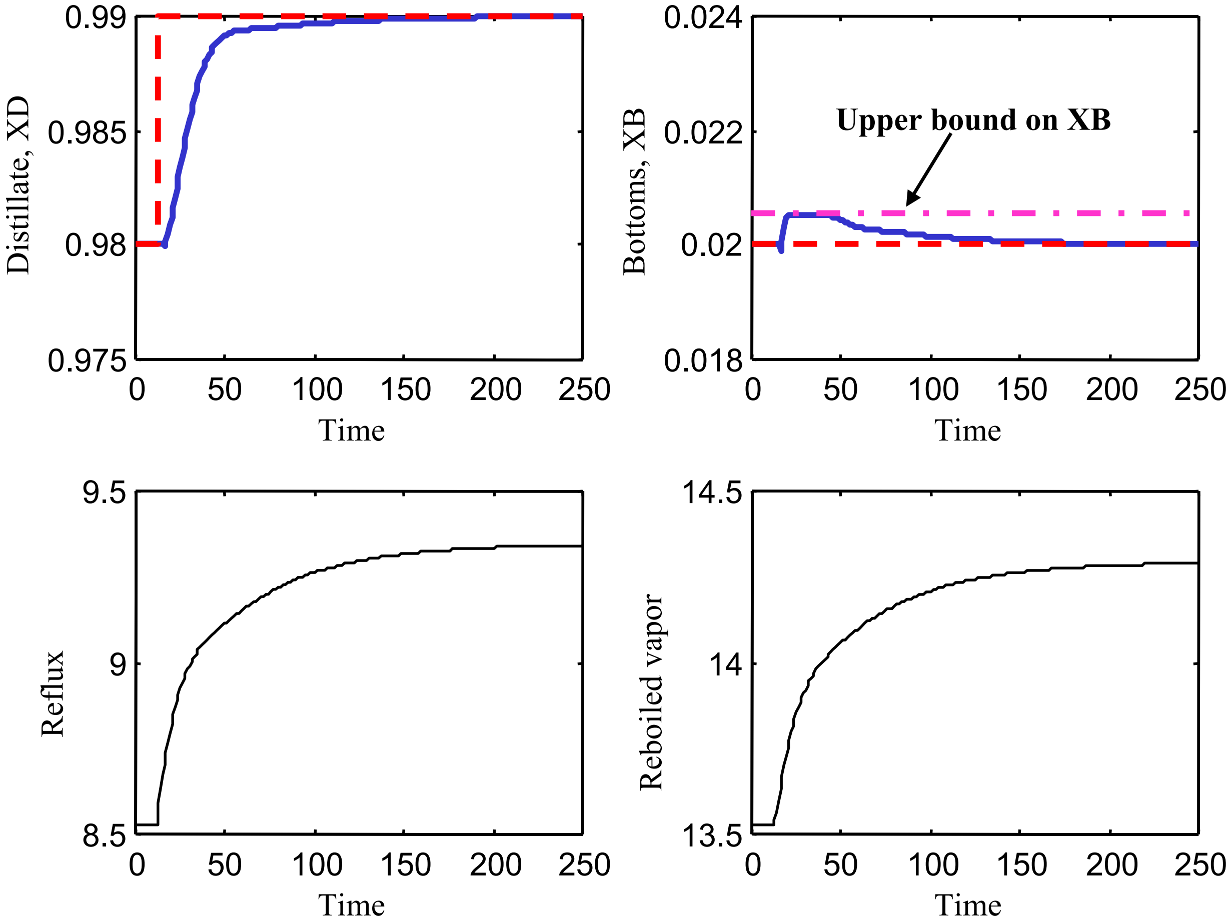
Discuss the control performance.





CHAPTER 23: Centralized MPC Control

Controller calculation: **Sample distillation without model error, move suppression, and bound on controlled variable.**



CHAPTER 23: Centralized MPC Control

KEY INSIGHTS

- The MPC design methods yields easily solved optimization problems, least squares (unconstrained, offline) or quadratic programming, QP (constrained, online)
- The constrained controller does not have a fixed control calculation; it changes in response to the active constraints
- The calculation of the QP is much more complex than typical controllers, but it gives extraordinary features.
  - The QP is convex, so that the global optimum is guaranteed at the solution.
  - Essentially every MPC includes the QP

CHAPTER 23: Centralized MPC Control

KEY INSIGHTS

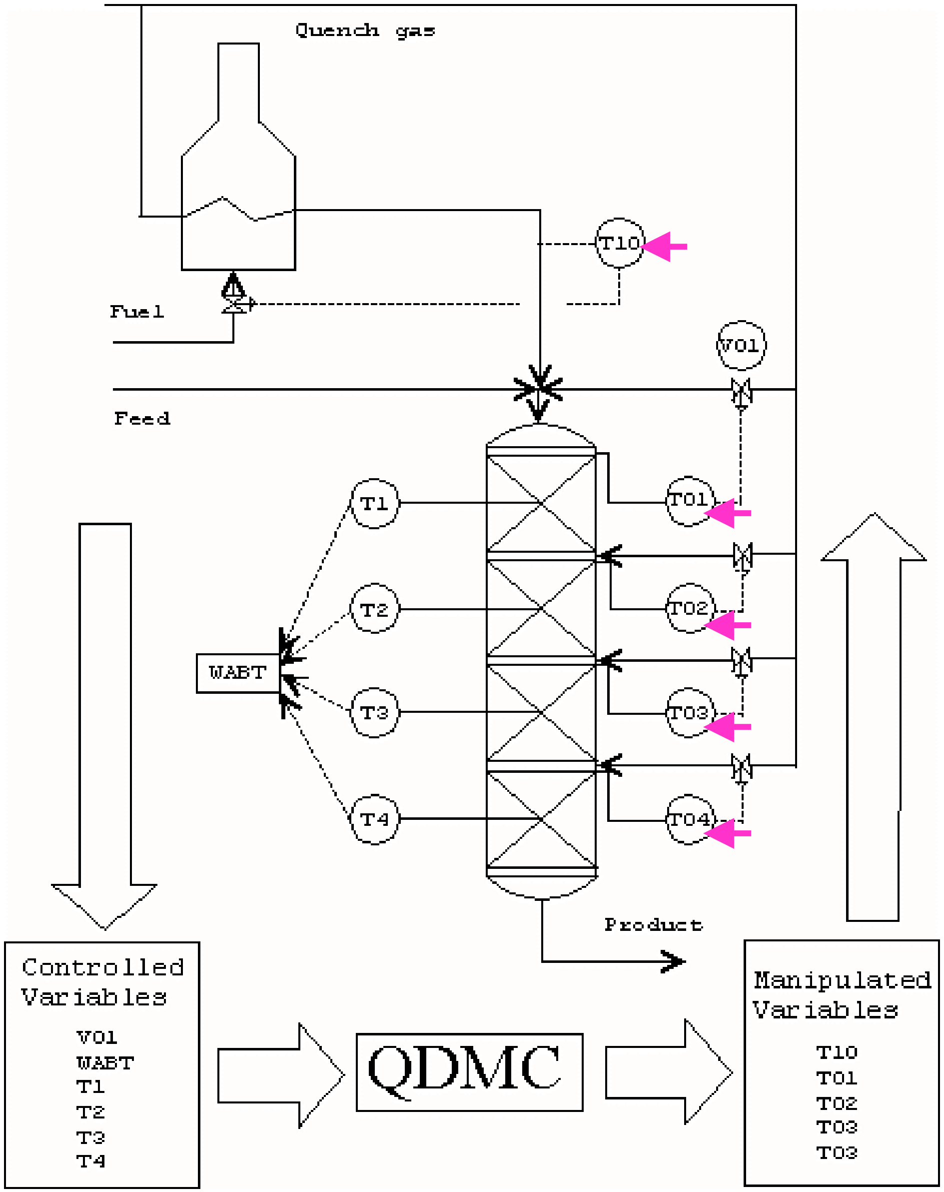
- **The MPC controller can easily be formulated for "non-square" systems, i.e. a different number of manipulated and controlled variable.**
- **The MPC controller can calculate the feedforward compensation for measured disturbances using the same, unmodified optimization. Only E^f is changed.**
- **The MPC can be used to display the predicted future transient to the operators.**
- **MPC controllers can handle large systems, certainty 10+ CVs and MVs, with much larger industrial applications reported**

CHAPTER 23: Centralized MPC Control

MPC APPLICATION GUIDELINES

- Use MPC for MIMO systems; SISO can use PID or IMC
- Non-square systems of high complexity, where signal select and split range become too complex
- Process systems with complex dynamics
- Process systems that often encounter constraints and the constraint activity changes
- MPC usually is arranged in a cascade structure resetting the set points of regulatory PID (or IMC) controllers
- See next slide for a small example of MPC

CHAPTER 23: Centralized MPC Control



OPERATING OBJECTIVES

- **Total "conversion" is key quality, measured by weighted average bed temperature**
- **Exothermic reactor beds must not "run away"**
- **Reaction in each bed depends on strategy, max selectivity or max catalyst life**
- **T10 should be low, but the bypass valve v01 (adjusted by T01) should be opened for fast control**

COMMERCIAL MPC PRODUCTS

- **Involve complex software, transparent to user**
- **Integrated empirical modelling and controller design**
- **Simulation test bed for controller tuning and evaluation**
- **Design and tuning on PC/Workstation with download of controller parameters to DCS**
- **Many other features not discussed here, e.g.,**
  - Steady-state optimization
  - Check for controllability and conditioning
  - Features for unstable processes (levels and other)
  - Enhanced feedback with disturbance model

CHAPTER 23: MPC CONTROL WORKSHOP 1

The MPC control objective involved the square of error (and move sizes). Discuss the advantages and disadvantages of using the power 2 (and not 1, 3, 4, 1/2, absolute value, etc.)

Hint: The choice is not arbitrary; it provides important advantages.

$$\min_{\Delta MV^c} \sum_{\substack{output \\ variables}} \sum_{samples=i} ww(E_i^f - CV_i^c)^2 + \sum_{\substack{manip. \\ variables}} \sum_{samples} qq(\Delta MV_i)^2$$

$$subject\ \ to$$

$$A\ \ \Delta MV^c = CV^c$$

CHAPTER 23: MPC CONTROL WORKSHOP 2

Describe the sequence of calculations performed in the offline design of MPC. Restrict your presentation to the unconstrained, multivariable MPC.

- **Begin with the empirical identification**
- **Include parameters that are manually inputted**
- **Organize the calculations in the order performed**

$$K_{DMC} = (A^T [WW] A + QQ)^{-1} A^T [WW]$$

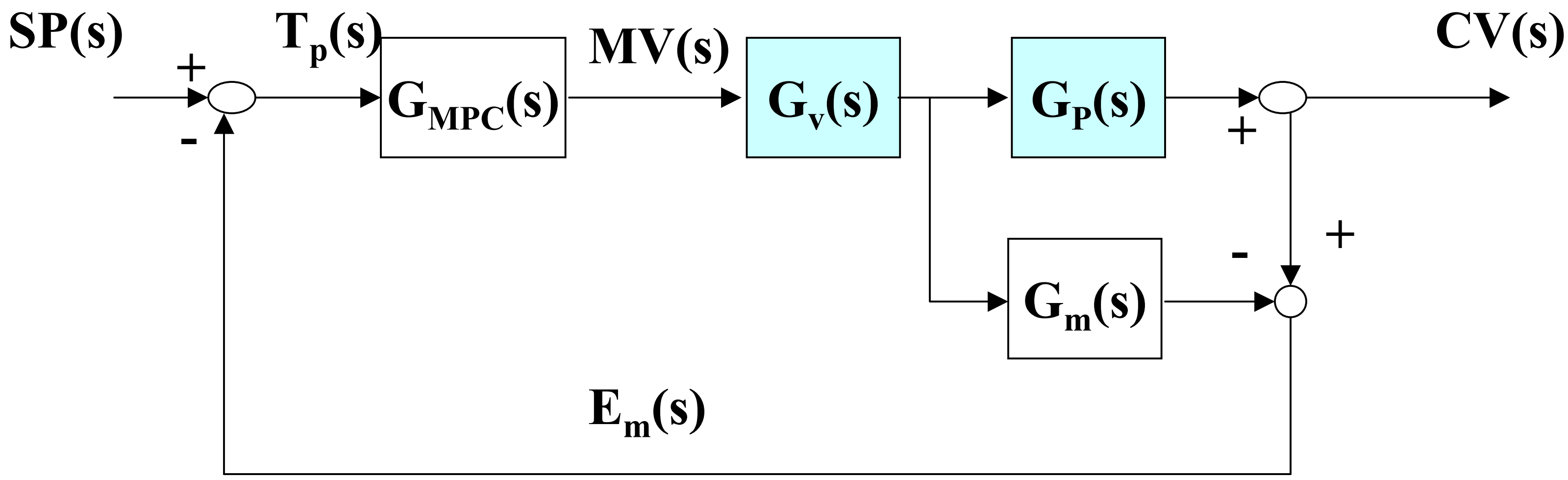
$$\left[\Delta MV^c\right] = \left[K_{DMC}\right]\left[E^f\right]$$

Solution is least squares equations, involving solution to linear equations offline and simple matrix-vector product online.

CHAPTER 23: MPC CONTROL WORKSHOP 3

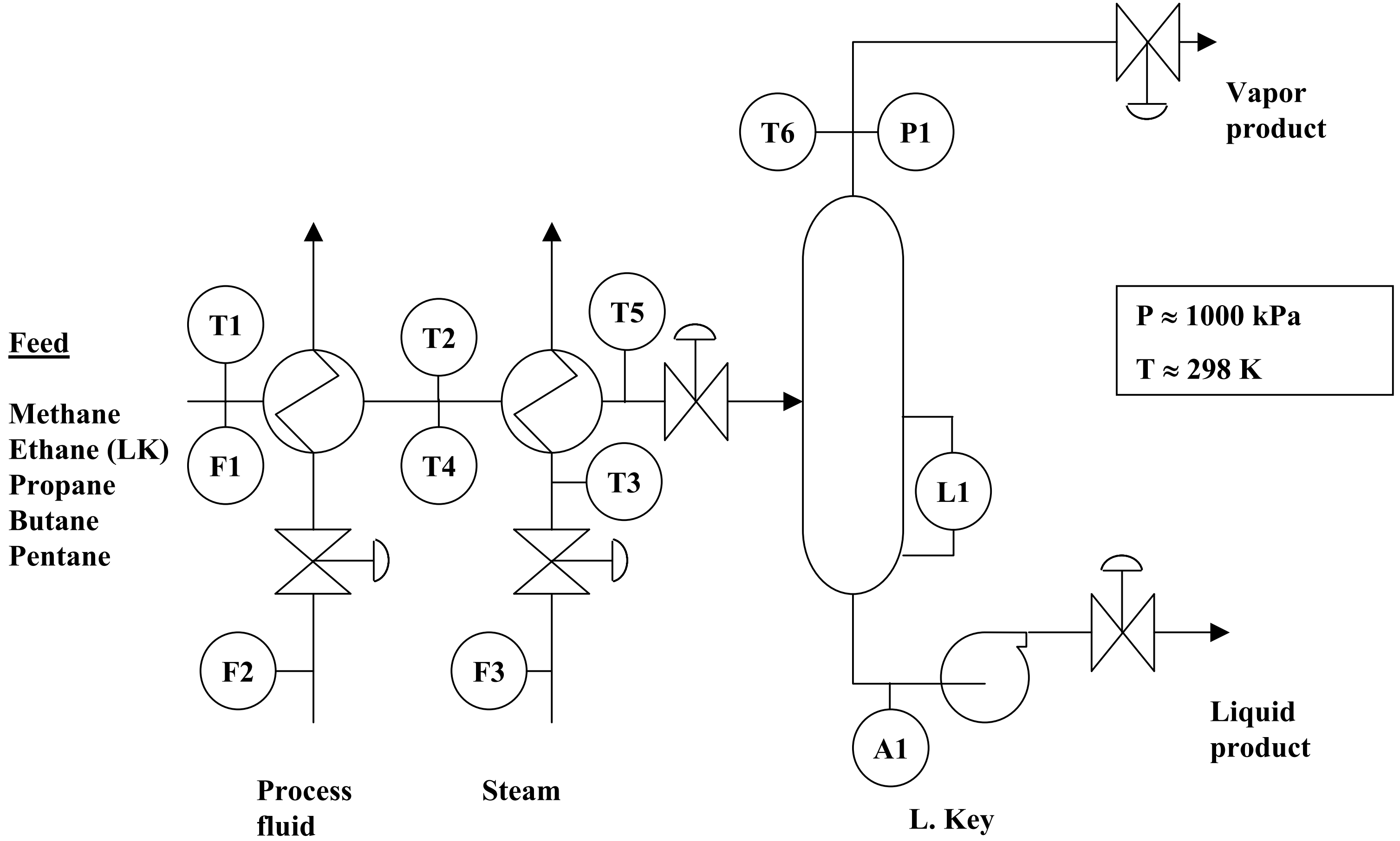
Describe the sequence of calculations performed by the MPC online controller at every execution.

- **Identify information from controller design, manual entry, and sensors**
- **Organize the calculations in the order performed**

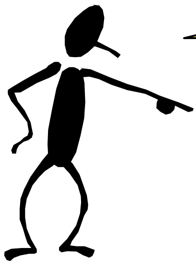


CHAPTER 23: MPC CONTROL WORKSHOP 4

Would you recommend applying multivariable MPC for control of the flash process?

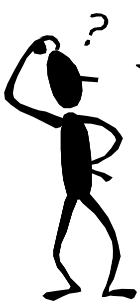


CHAPTER 23: Centralized MPC Control



When I complete this chapter, I want to be able to do the following.

- **Explain centralized control**
- **Build a discrete process model**
- **Explain the trajectory optimization for MPC**
- **Explain the advantages of MPC**
- **Describe criteria for selecting MPC over multiloop control**



Lot's of improvement, but we need some more study!

- **Read the textbook**
- **Review the notes, especially learning goals and workshop**
- **Try out the self-study suggestions**
- **Naturally, we'll have an assignment!**

CHAPTER 23: LEARNING RESOURCES

- **<u>SITE PC-EDUCATION WEB</u>**
  - Tutorials (Chapter 23)

- **The Textbook, naturally, for more examples.**

- **Addition information on MPC control is given in the following reference.**

Brosilow, C. and B. Joseph, *Techniques of Model-Based Control*, Prentice Hall, Upper Saddle River, 2002 - Some additional introductory material on MPC

Maciejowski, J.M., *Predictive Control with Constraints*, Prentice Hall, Harlow, England, 2002 - A clearly presented, comprehensive presentation at the advanced level

Qin, S. J. and T. Badgwell - A Survey of industrial model predictive control technology, *Control Engineering Practice*, 11, 733-764, 2003

CHAPTER 23: SUGGESTIONS FOR SELF-STUDY

1. **Discuss the similarities and differences between IMC and MPC algorithms.**
2. **Find a recent journal article with an industrial application of MPC. Describe why you think that MPC was a good (or poor) choice for control.**
3. **Prove that the solution to the MPC controller gain (K_{MPC}) is the least squares equation.**
4. **Discuss the effect of one or more manipulated variable encountering and remaining at a bound (either upper or lower).**
5. **Compare PID with decoupling with multivariable MPC; discuss advantages of each.**